MATHEMATICAL SCIENCES

AMMIN/DISMA - Mathematics of porous media and metamaterials, inspired by the mechanics of nonholonomic systems: theoretical, constitutive, and computational aspects

Funded By	Dipartimento DISMA Politecnico di TORINO [P.iva/CF:00518460019]

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Context of the research activity	The focus of this research project is the detailed mathematical description of porous media and metamaterials in the fields of biomedicine and environmental sciences. The project aims at improving the understanding and the mathematical formalization of the behavior of such media through the formulation of constitutive theories capable of resolving mechanical, chemical, thermal and electromagnetic interactions, starting from the knowledge of the internal structure of the considered materials.

	An evergreen topic in Applied Mathematics is the formulation of models and computational methods aimed at describing materials of interest in biomedicine, industry, and environmental sciences. While aggregates of cultured cells and biological tissues grown in vitro are employed to analyze experimentally living matter, engineered and bio-inspired materials attract attention for their use in biomedicine and transplant surgery. To be valid substitutes of biological media, these artificial materials must be constructed so as to best match the physiological requirements that native tissues fulfill. In this respect, metamaterials –manufactured materials conceived to have a well-defined function– offer a wide variety of solutions for different problems, such as the guidance of “stem cells” [Munding et al., 2023], the optimization of “drug delivery” [Xu et al., 2023], and the design of “biomedical antennas” [Tzarouchis et al., 2024]. In the industrial world, metamaterials find applications in optics and acoustics, for instance in the contexts of the absorption of light [Wang et al., 2024] and sound waves [Krushynska et al., 2017; Arjunan et al., 2024], while in the environmental sciences they are used, e.g., in the attenuation of the damages generated by earthquakes [Maheshwari and Rajagopal, 2022; Tan et al., 2025]. Since metamaterials do not exist in nature, they have to be designed to meet the necessities dictated by their use. To this end, it is convenient to represent them with equivalent continua, the effective properties of which are obtained

Objectives	through averaging or homogenization procedures that up-scale the geometric, topological, and material properties of their internal structure. For the effective properties to be truly descriptive, the mathematical procedures leading to their determination must be accurate and precise. This is the main motivation for intensifying the research in this field and the reason for providing a PhD scholarship funding further studies. The aim of these studies is to improve the already existing mathematical methods and to formulate new ones. Consistently with the scopes outlined above, this project focuses on porous media and metamaterials with periodic internal structure, obtained by repeating a representative “cell”. In detail, the project proposes four milestones: 1. Following a research line started by Bigoni and co-workers [Cazzolli, 2020; Bigoni, 2023], and in collaboration with the research group of Prof. Davide Bigoni (University of Trento, Italy), we will consider metamaterials in which the internal structure of the representative cell features a discrete or a continuum mechanical system, possibly subjected to nonholonomic constraints. We will study the stability of such materials and, after homogenization, the material symmetries of the equivalent continua. 2. We will address the case in which the discrete mechanical system under consideration comprises devices that obey response functions of fractional type. In this case, the goal is the determination of the effective fractional behavior of the equivalent continuum through the up-scaling of the nonlocality introduced at the level of the representative cell. 3. We will consider metamaterials in which the representative cell consists of a porous medium, saturated by one or more fluids, that can be described with the aid of “Hybrid Mixture Theory” [Bennethum et al., 2000] even at the scale of the internal structure. This type of materials is of interest, e.g., for the construction of bio-inspired devices and engineered tissues. 4. We study the electromagnetic properties of metamaterials and porous media. Our goals are: (i) investigating the propagation of electromagnetic waves in such materials; (ii) investigating how the effective electromechanical properties of their equivalent continua can be influenced by the polarization or magnetization of the constituents of the representative cell. Funding: “Finanziato dall’Unione europea-Next Generation EU, Missione 4 Componente 1 CUP P2022KHFNB”, within the research project: “Innovative multiscale approaches, possibly based on Fractional Calculus, for the effective constitutive modeling of cell mechanics, engineered tissues, and metamaterials in Biomedicine and related fields”, PRIN PNRR P2022KHFNB. References Arjunan, A., et al., Bulding Environ, 2024; 251: 111250 Bennethum, L.S., et al., Transport Porous Med, 2000; 39(2): 187–225 Bigoni, D., et al., Proc R Soc A, 2023; 479: 20230523 Cazzolli, A., et al., J Mech Phys Solids, 2020; 138: 103919 Krushynska, A.O., et al., Acta Acust United Ac, 2018; 104: 200–210 Maheshwari, H.K. and P. Rajagopal, Soil Dyn. Earthq. Eng., 2022; 161: 107409 Munding, N., et al., Adv. Func. Mater., 2024; 34: 2301133 Tan, J., et al., Physics Letters A, 2025; 545: 130505 Tzarouchis, D.C., et al., EPJ Appl. Metamat., 2024; 11(7)

	Wang, B.-X., et al., Adv. Func. Mater., 2024; 34: 2402068 Xu, J., et al., Nat. Commun., 2023; 14: 869

Skills and competencies for the development of the activity	The candidate should have worked, or should be currently working, on the mechanics of porous media at the level of the Master of Science thesis. The candidate, in addition, should have some experience with the most fundamental tools of Analytical Mechanics, Variational Calculus, Differential Geometry, and Finite Element Methods. The candidate should also be willing to perform numerical simulations either with proprietary or with open-source software.