Politecnico di Torino

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MATHEMATICAL SCIENCES

PNRR/FAIR - Morse Homology, Morse-Smale Complexes, Morse Theory, and Their Relationship with AI

Funded By	MINISTERO DELL'UNIVERSITA' E DELLA RICERCA [P.iva/CF:97429780584] Politecnico di TORINO [P.iva/CF:00518460019]

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Context of the research activity	This project explores the intersection of Morse theory, Morse homology, and artificial intelligence (AI). By leveraging the rich topological insights from Morse theory, the study aims to investigate how homology can enhance machine learning models by uncovering hidden structures in high-dimensional data, bridging abstract mathematics and practical AI advancements. Progetto finanziato nell'ambito del PNRR - M4C2, Investimento 1.3 - Avviso n. 341 del 15/03/2022 - PE0000013 Future Artificial Intelligence Research (FAIR) - CUP E13C22001800001.

	This project delves into the synergy between Morse theory, Morse homology, and artificial intelligence (AI), aiming to harness the topological insights of Morse theory to enhance machine learning. The work bridges abstract mathematical frameworks and practical AI applications by investigating how homology can uncover hidden structures in high-dimensional data. Morse homology and Morse-Smale complexes, rooted in studying smooth manifolds and critical points of differentiable functions, offer powerful tools for understanding the topology of high-dimensional spaces. These concepts have emerging applications in machine learning, image processing, and artificial intelligence, where data is often represented in complex, high-dimensional forms. Here are key intersections: - Morse theory provides a framework for analyzing the critical points of loss functions in machine learning. Understanding the topology of these critical points can offer insights into the optimization landscape, helping develop better training algorithms. For instance, Morse-Smale complexes can partition the parameter space into regions of similar behaviour, offering a structured way to navigate optimization pathways. Persistent homology, often computed using Morse theory, can extract topological features from data, aiding in robust feature selection and

Objectives	dimensionality reduction. - Feature Detection in Image Processing In image processing, Morse-Smale complexes provide a natural way to analyze scalar fields, such as intensity maps of images: The Morse-Smale complex decomposes an image into regions based on gradient flows between critical points, enabling hierarchical representations. These decompositions are particularly useful for keypoint detection, edge detection, and segmentation tasks, as they are invariant to transformations and robust to noise. - Topological Data Analysis in AI Morse homology underpins many tools in topological data analysis (TDA), which is increasingly used in AI to analyze data shape. Persistent homology, derived from Morse functions, is used to study the multi-scale structure of data. This has applications in anomaly detection, clustering, and generative modelling. TDA methods inspired by Morse theory can identify structural patterns invariant to deformations in image classification, improving model generalization. - Advancing AI Models with Morse Theory The gradient flows, and critical point structures of Morse-Smale complexes can inspire novel architectures and algorithms: Loss functions inspired by Morse theory may smooth optimization landscapes, reducing the risk of getting trapped in poor local minima. In neural networks, Morse-based priors can impose topological constraints, leading to models that better capture the intrinsic structure of data. Medical Imaging: Morse-Smale complexes are applied to segment and analyze anatomical structures in 3D scans, providing insights into critical features such as tumour boundaries or vascular networks. Robotics: In path planning, Morse theory helps analyze and optimize potential field landscapes, guiding robots through complex terrains with fewer computational resources. Conclusion Morse homology and Morse-Smale complexes bridge the abstract world of differential topology and practical challenges in machine learning and image processing. By capturing data's intrinsic geometric and topological structures, these methods promise more robust, interpretable, and efficient solutions to complex AI problems.

Skills and competencies for the development of the activity	The ideal candidate has a degree in mathematical engineering and a strong and verifiable background in applied topology, machine learning, deep learning, and programming.