Politecnico di Torino

MATHEMATICAL SCIENCES

DISMA - Advanced numerical methods for wave propagation problems

Funded By	Dipartimento di Scienze Matematiche [P.iva/CF:00518460019]
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Context of the research activity	The subject of the research is the numerical simulation of wave propagation problems, which are described by Partial Differential Equations (PDEs) and solved using interior domain and Boundary Element Methods (BEMs). Specifically, the focus is on the innovative Virtual Element Method (VEM) for three-dimensional problems defined in bounded domains, combined with a BEM one for addressing exterior problems.
Objectives	The numerical solution of evolution problems, such as acoustic and elastic wave propagation, has been a significant area of research for many years. Nowadays, it stands as one of the most crucial challenges in scientific computing. Indeed, the study and analysis of wave propagation are relevant across various fields of applied sciences, including acoustics, elastodynamics, fracture mechanics, geophysics, fluid dynamics, and meteorology. Typically, the phenomenon of interest is localized within a vast surrounding medium. Although the exterior region may not be strictly unbounded, the boundary effects are often negligible. As a result, the problem can be simplified by replacing the extensive exterior with an infinite medium. This approach leads to the study of problems described by Partial Differential Equations (PDEs) defined on exterior domains, which require handling the issue of unboundedness in numerical simulations. When dealing with problems involving constant coefficients, a useful approach is to reformulate the PDE in terms of a Boundary Integral Equation (BIE) and solve it using a Boundary Element Method (BEM). The BEM utilizes the fundamental solution associated with the differential problem. Alternatively, when the material properties of the exterior domain vary, domain discretization methods such as the classical Finite Element Method (FEM), Finite Difference Method (FDM), or Finite Volume Method (FVM) can be employed. In this case, after defining the bounded computational domain for studying the solution behavior, introducing proper (transparent) boundary conditions becomes a crucial step. These conditions must ensure that the solution of the initial boundary value problem inside the finite computational domain coincides with the restriction of the solution of the original problem. This approach involves a coupling strategy, where the domain method

	serves as a solver within the finite computational domain, while the BEM acts as a Non-Reflecting Boundary Condition (NRBC). The objective of the PhD scholarship is to develop innovative and efficient numerical methods for approximating the aforementioned wave propagation problems. Special emphasis will be placed on integral and integro-differential models related to the propagation of vector-type problems, such as electromagnetic or elastic waves, in two- and, especially, three-dimensional unbounded domains. To address these problems, we aim to apply novel strategies in both the pure BEM approach and its coupling with a domain method. In the latter case, we will utilize the Virtual Element discretization Method (VEM). VEMs have recently been introduced and successfully applied in solving problems defined in bounded regions, enabling an expansion of the classical family of FEMs for discretizing PDEs. VEMs offer advantages in terms of the decomposition of the computational domain and the definition of local discrete spaces, leading to benefits in computational efficiency and implementation. Due to these advantages over standard FEMs, our goal is to employ VEMs coupled with BEMs to solve exterior 2D/3D evolution problems described by PDEs, both in the frequency or time domain.

Skills and competencies for the development of the activity	Advanced knowledges in Numerical Analysis and computational methods, variational formulation of PDEs, FEM methods, meshing techniques. Programming skills and knowledge of Matlab/ C/ C++ programming languages.