Politecnico di Torino

CIVIL AND ENVIRONMENTAL ENGINEERING

DISEG/CRT - Phase-Field and Fractional Calculus modelling in the framework of Solid and Fracture Mechanics

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Context of the research activity	The work will be predominantly numerical and will consist of writing efficient algorithms, both in the field of finite elements and in the more general one of variational approaches. The main objectives consist in modelling crack propagation in structures presenting cracks and defects, also studying the interaction between stress raisers, and in setting and solving fractional differential equations that describe the static and dynamic equilibrium in nonlocal solids.

	Two topics are herein described: Phase Field (PF) modelling and Fractional Calculus (FC). PF models have emerged as a highly effective solution approach for issues related to phase boundary evolution during phase transformations. Initially designed for describing phase changes, these models have been adapted for addressing concerns related to electrochemical reactions, mechanical deformations, and fracture processes. The elegance of PF formulations in handling these moving boundary problems lies in the absence of explicit treatment of boundary conditions at the dynamic interface. As a result, complex morphological changes can be readily applied across various dimensions. In the framework of fracture mechanics, let us mention the following applications: i) Dynamic fracture: PF modelling enables the study of fracture propagation in materials under dynamic loading, such as in cases of impact or shock; ii) Fracture in complex materials: this technique is useful for addressing fracture in complex materials, such as composites, porous materials, or multilayers; iii) Interaction with defects: PF modelling enables examining how fracture interacts with various defects, like inclusions or pre-existing cracks. On the other hand, FC is a mathematical discipline that deals with integrals and derivatives of non-integer order. Its applications in solid mechanics are relatively recent and primarily associated with the behaviour of viscoelastic

Objectives	materials (i.e., fractional damping). Research involving fractional spatial derivatives is even more recent and voted to investigate a material with nonlocal stress interactions defined through the fractional integral of the strain field. One distinctive feature of fractional nonlocal elastic materials is the long tails of the power law kernel in fractional integrals, which give rise to unique mechanical behaviours. Additionally, these materials exhibit power law size effects due to the anomalous physical dimension of fractional operators. Furthermore, it has been demonstrated that the fractional nonlocal elastic medium can be regarded as a continuum limit of a lattice model, with points connected by three levels of springs, and stiffness decaying according to a power law based on the distance between connected points. Importantly, the fractional model takes into account interactions between bulk and surface material points distinctly. The fractional differential equation governing the equilibrium, along with the appropriate static and kinematic boundary conditions, was solved through fractional finite differences in terms of the displacement function using a suitable numerical algorithm. It is important also to underline that the model present above can be also applied to investigate dynamic problems such as wave propagation and diffusion processes, by properly considering the contribution of a time derivative. In the framework described above these are the goals which have been identified: 1. To minimize the computational effort of PF modelling required for crack propagation studies, optimizing the adopted meshes and the models' implementation through efficient computing platforms. 2. To implement a machine learning assisted PF modelling, to improve computational efficiency and accuracy, and to enable the prediction of fracture behaviour under more complex loading and environmental conditions. 3. To generalize the fractional model proposed in the field of solid mechanics to the two-dimensional case, solving the problem by implementing (fractional) finite elements. 4. To investigate the FC potentialities in the context of the crack onset and crack propagation modelling, through the discussion and generalization of pertinent theorems about models' analytical properties and their implementation on proper computing platforms.

Skills and competencies for the development of the activity	The following skills are required: - Excellent knowledge of the theoretical fundamentals of linear elastic fracture mechanics. - Solid background in the framework of numerical calculus. In particular, a good knowledge of structural FEM softwares is required. The knowledge of MATLAB is also useful. Furthermore, the candidate is required: - excellent attitude to work in a team. - availability to spend a period abroad (at least 3 months) to perform part of the modelling work.