MECHANICS REVIEW

Summary

DRAWING: INTRODUCTION

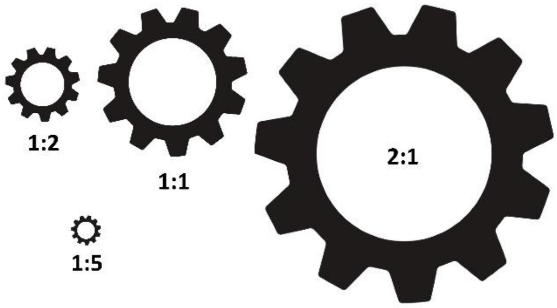
The purpose of this paper is to provide a guideline of the basic knowledge in engineering drawing necessary to follow the Mechatronics Master Degree. All the material should not be considered as sufficient to provide a complete knowledge, but as a guideline to the students that are approaching to the courses and that want to review their knowledge or to fulfill their lack.

SCALE, LINES, SIZES

To fully describe an object, the drawer must provide some basic information.

One of the most important is the ***scale*** of the drawing, that indicates the ratio between a length in the drawing and the same length the real world.

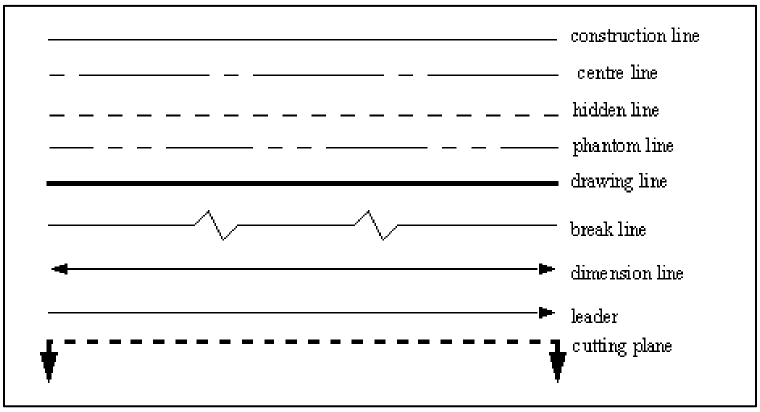
The scale is very important because enables the measurements to be read.



Another important information is provided by the kind of ***line*** used by the drawer.

There are many different types of lines, used in different situations:

Line	Description	Application
visible	continuous	Edges directly visible
hidden	short-dashed	Edges not directly visible
centre	alternately long- and short-dashed lines	Symmetry axes
cutting plane	medium-dashed	Sections
section	thin	surface from section
phantom	alternately long- and double short-dashed thin	Component not part of the main object

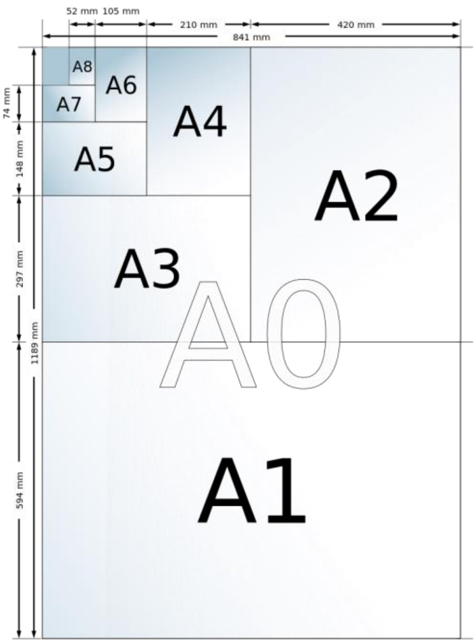


Another way to classify the lines, according to the standard, is using the letters as in the following picture:

Line		Description	General Application	
A		Continuous thick	A1 A2	Visible outlines. Visible edges.
B		Continuous thin (straight or curved)	B1 B2 B3 B4 B5 B6 B7	Imaginary lines of intersection. Dimension lines. Projection lines. Leader lines. Hatching lines. Outlines of revolved sections in place. Short centre lines
C		Continuous thin free hand	C1	Limits of partial or interrupted views and sections, If the limit is not a chain thin.
D		Continuous thin (straight) with zigzags	D1	Long break line
E		Dashed thick	E1 E2	Hidden outlines. Hidden edges.
F		Dashed thin	F1 F2	Hidden outlines. Hidden edges.
G		Chain thin	G1 G2 G3	Center lines. Lines of symmetry. Trajectories
H		Chain thin, thick at ends and changes of direction	H1	Cutting planes.
J		Chain thick	J1	Indication of lines or surfaces to which a special requirement applies
K		Chain thin double dashed	K1 K1 K3 K4 K5	Outlines of adjacent parts. Alternative or extreme position of movable parts. Centroidal lines. Initial outlines prior to forming Parts situated in front of the cutting plane

The last information is related to the ***size of the drawing*** that, according to the ISO standard are:

A0	841	1189
A1	594	841
A2	420	594
A3	297	420
A4	210	297



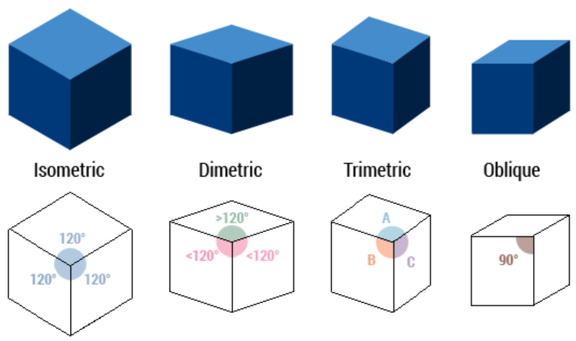
PROJECTIONS

To represent physical objects in a two-dimensional plane it is necessary to apply a protocol by which the image of the three-dimensional subject of the study is projected in a planar space: this is call projection.

There are two main graphical families, with many different kinds of projections regulated by its own general rules. The following scheme is a resume of the main projection categories with some example.

- **I. PARALLEL PROJECTION**
  - a. Pictorials
    - i. Axonometric Projection
      1. Isometric
      2. Dimetric
      3. Trimetric
  - b. Oblique Projection
  - c. Orthographic Projection or **Multi-View** Drawing
      1. First angle (Primo diedro): ISO Standard
      2. Third angle (Terzo diedro): ASME Standard
- **II. PERSPECTIVE PROJECTION**

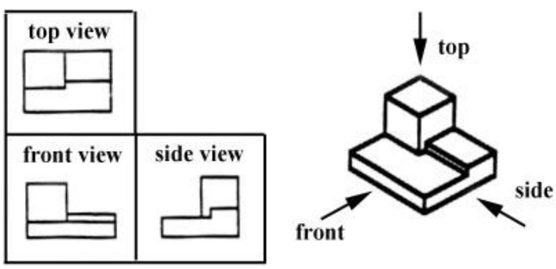
The *pictorial* is the family of the three-dimensional drawing that contains one of the most common way of sketching, the ***isometric drawing***. In this kind of projection, the scales of the two axes is the same and the angle between them is 120 degrees. In this way, the object's vertical lines remain vertical in the planar plane and the other two are represented by an angle of 30 degrees to the horizontal, maintaining constant the different lengths and enabling measurements to be read or taken directly from the drawing.



This represents the base for 3D sketches and it is commonly used.

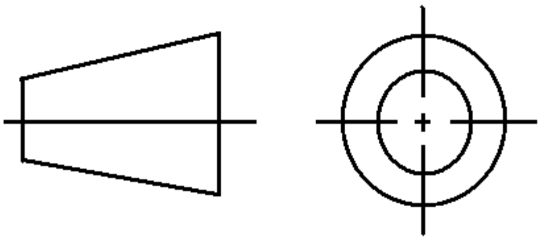
Another important family of projections is the ***orthographic*** (or ***multi-view***), that allows to represent all the information related to an object, putting it inside an ideal "glass box" and projecting each face of it to the corresponding size of the box.

This generates six principal views (FRONT, TOP, LEFT and RIGHT SIDE, BOTTOM and REAR) but generally, only three of these are needed to completely describe the object. We need as many views as are required to fully described the object, in a simple and economy way: sometimes the number of necessary views can be less than three (as in the case of a cylinder or a shaft) but sometimes it can be necessary to add an *auxiliary projection* (a projection in a plane different from the six principal view).

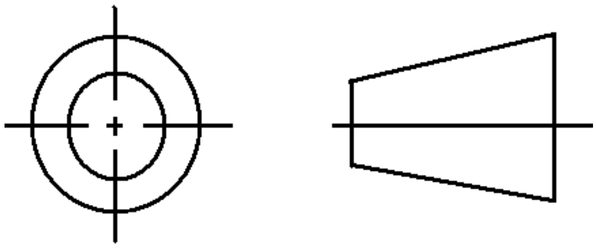


The position of the principal view one each other is determinate according to one of this two projection rules:

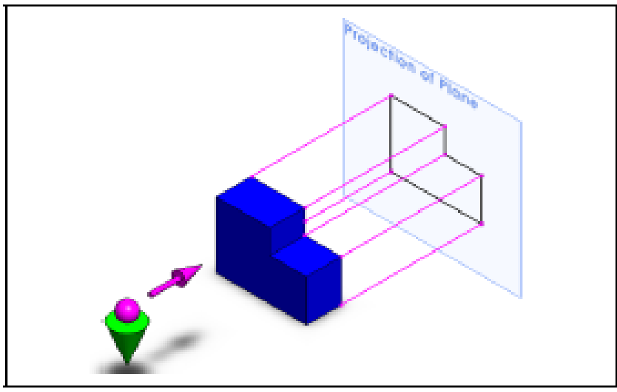
- ***FIRST-ANGLE*** projection, used in the ISO Standard (Europe), project the object in the same direction of an external observer, in the plane *behind* the object. In this case, TOP view is pushed down to the floor and FRONT is pushed back to the rear wall of the box;
- ***THIRD-ANGLE*** projection, used in the ASME STANDARD (North America), project the object in the opposite direction of an external observer, in the plane *in front of* the object. In this case, TOP view is pushed up to the ceiling and FRONT is pushed to the front wall of the box.



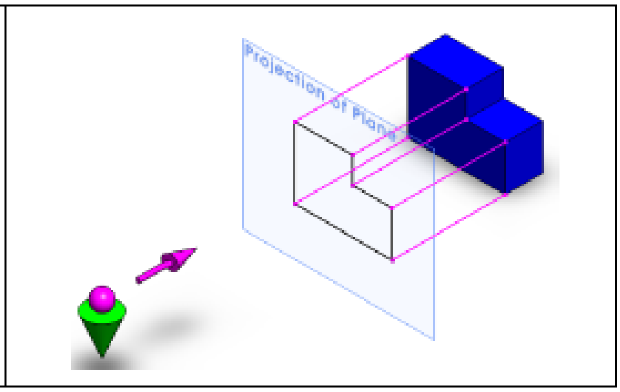
First angle Symbol



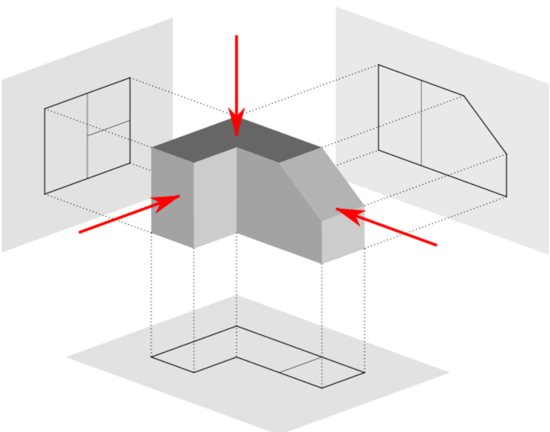
Third angle Symbol



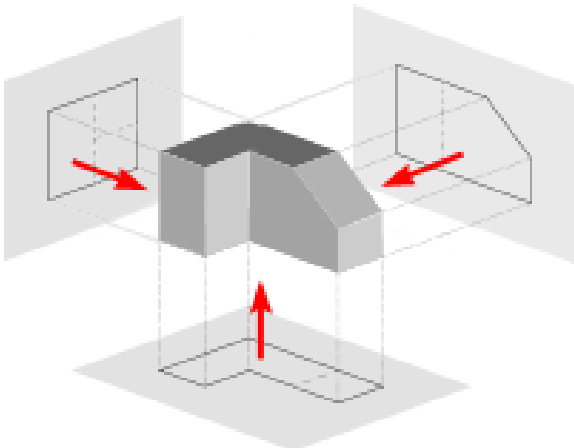
First angle view



Third angle view



First angle projection

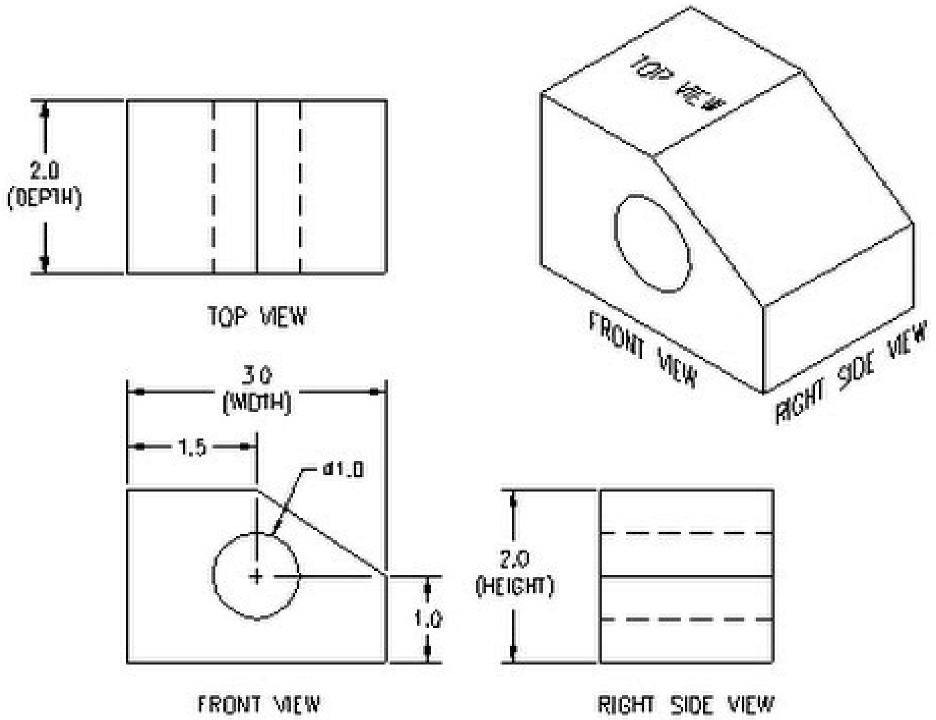


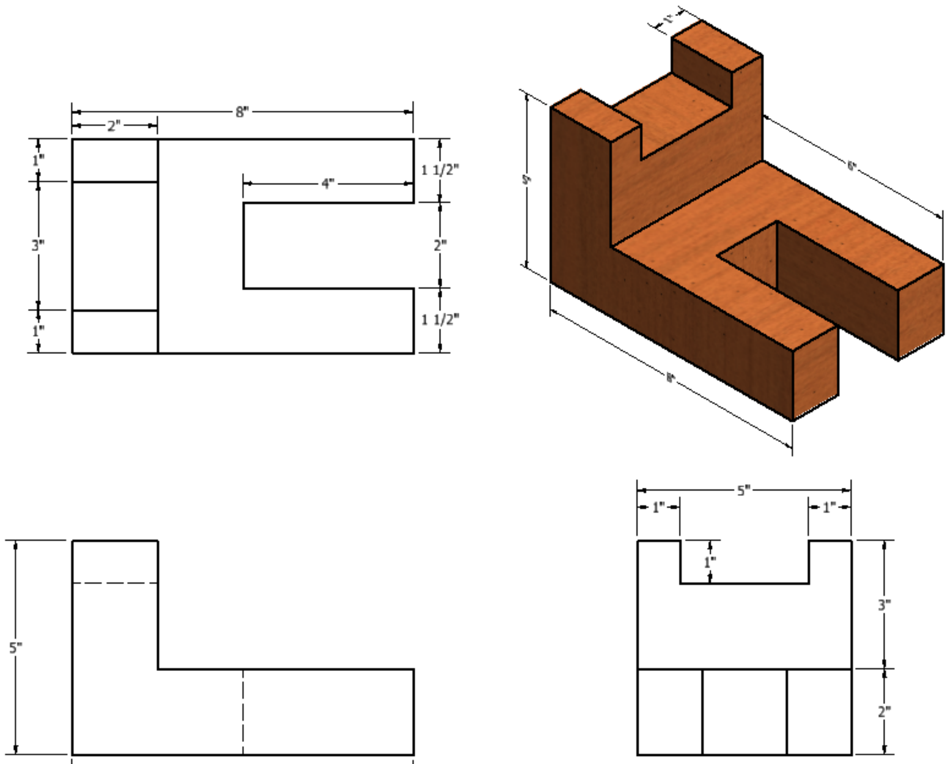
Third angle projection

MECHANICS REVIEW

REMARK: dashed thick lines used to represent a hidden part.

EXAMPLES
In these cases, the dimensioning of the drawing is present (the topic is present in the following chapters).





Basic Technical Drawing, 8th by Spencer, Fig. 7-27 #1, p. 138

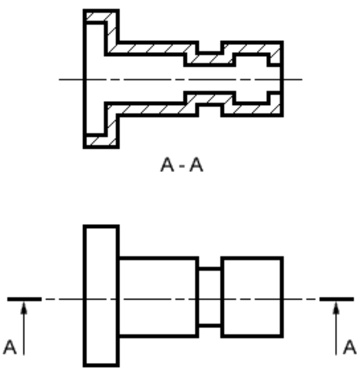
SECTIONS

Usually, it is not always possible to completely and clearly represent an object using different view: holes, hidden shapes or element can be present. The draw in these situations could be not clear for the presence of too many hidden lines.

The section is the representation, using the orthogonal projections, of one of the parts of the object, divided using one (or more) ideal plane, to show hidden components or hidden shapes in the device.

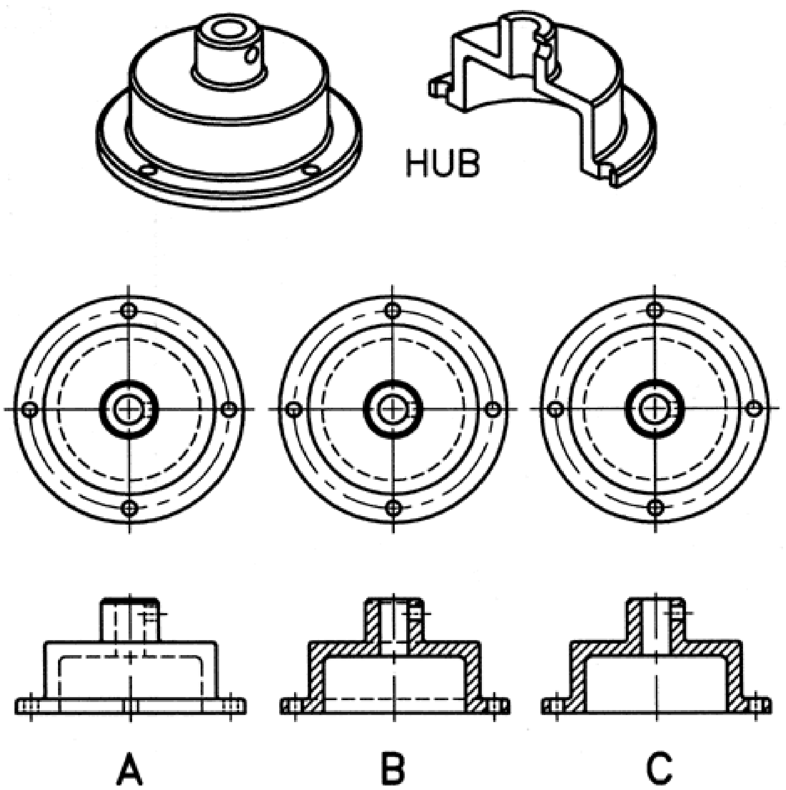
The sections help the drawer to use the minimum number of view and to provide all the information in a powerful way, avoiding the use of hidden lines that could make the drawing too complex to be read.

In any case, there always be a reason to use a section, that must always add information to the drawing.



As we can see in the example on the right, a very simple section provides all the information about the geometry of the component. In this case, using a common projection (frontal view) a lot of hidden lines were necessary to describe, in a less efficient way, the same geometry.

The direction of the projection is determinate using the arrows that indicate the name of the cutting plane, that is the same of the section (as in the example "A-A").



Here another example to underline these concepts.

The figure shows three different ways to represent the same object.

As we can see, in the projection **A** no cross section is used: the geometry of the object is difficult to understand due to the high number of hidden lines.

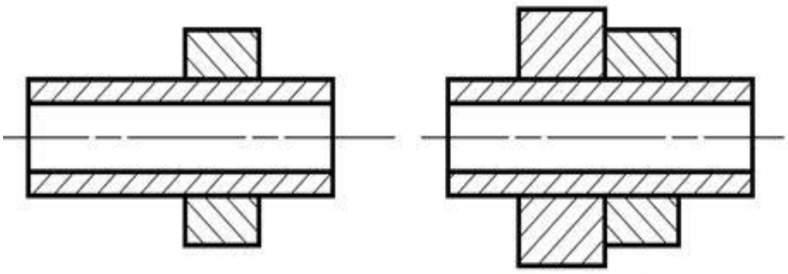
In the case **B,** even if a cross section is present (along with the horizontal axis), the presence of hidden lines does not add any new information to the draw.

This is the main reason why the last view **C** is the best one to describe the object: the geometry is completely described in an effective and clear way.

It can be noticed that the axis of the symmetric elements (in this case the four cylindrical holes) are still represented in the cross section, using thin long/short alternately lines. These lines cannot be avoided inside the draw or the resulting geometry could be ambiguous.

To complete the draw in the **B** and **C** cases the section line must be indicated, in the same way, used in the previously example (using arrows, thick ended line and named the cross-section) to avoid misunderstanding.

The *cross-hatches* represent the region where the material have been cut by the cutting-plane. Usually, these are diagonal lines, with an angle of 45° (but in general depends on the material) and have the same inclination for parts that belong to the same object. The used of thin lines is useful to underline the presence of different parts, or material, depending on the inclination of the line and by the kind of line.



Here an example of two different cross section.

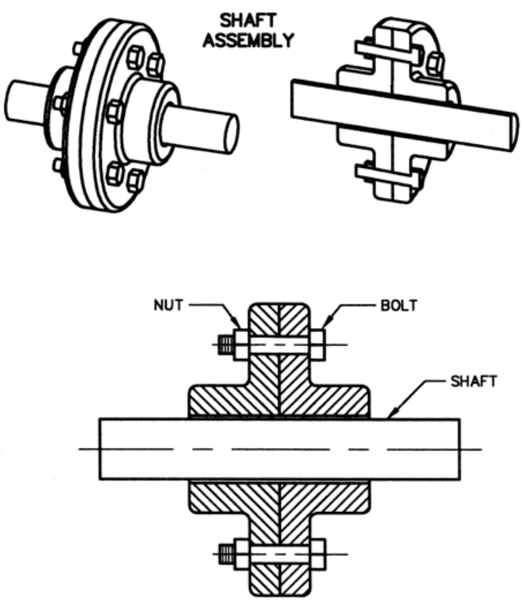
In this case the left object present two components, as we can see from the presence of two different line inclinations.

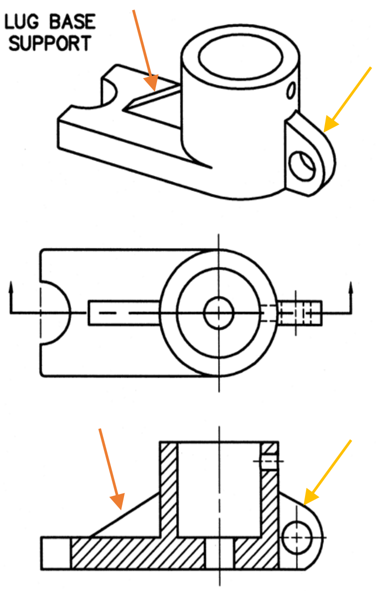
Instead, the right object is made assembling three different components: this information is "hidden" in the three different inclinations of the thin lines.

The section presents some exceptions (some parts that cannot be represented as "cut"):

- components "thin" respect to the bigger dimension, that is parallel to the cutting plane
- bearings
- nuts
- screws
- pin

Here two examples to better clarify these concepts.



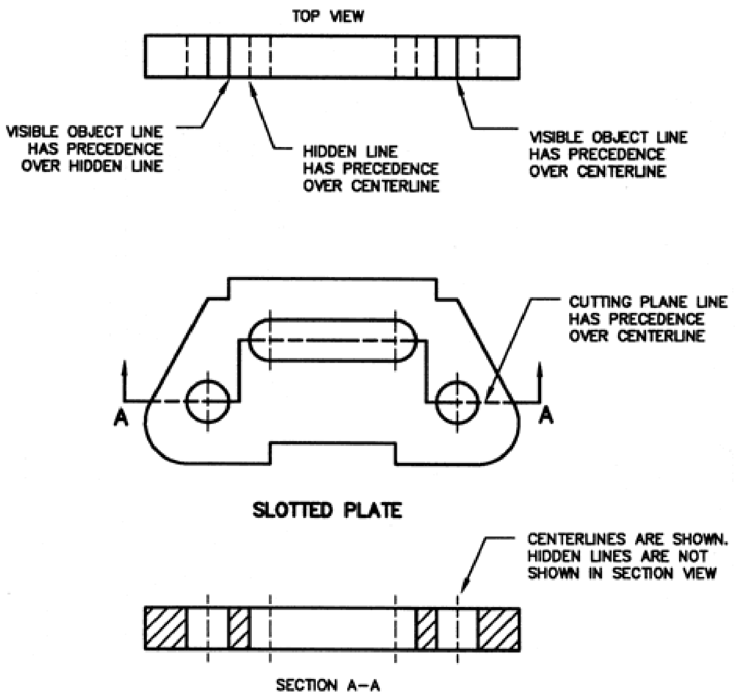


As we can see in the figure on the left, the screws and the shaft are not sectioned. Moreover, due to their cylindrical shape, they are represented by their symmetrical axes. Another important concept that can be noticed in this drawing is the presence of two different components, assembled together using some screws, thanks to the different orientation of the thin lines.

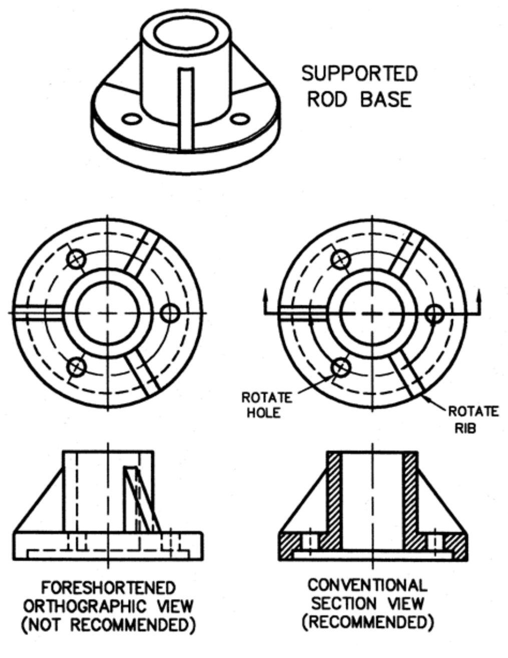
In the right figure, we can see how two central elements (the L-part on the left and the extension with the hole on the right, indicated by two arrows) are not sectioned. This is because they are considered "thin" respect to the dimension parallel to the cutting plane, in this case, the length of the component. Moreover, we can see how the cross-section line coincides with the symmetry axis of the component and with the plane that contains the main hole axis, to simplify the object representation.

Furthermore, sections can be made using:

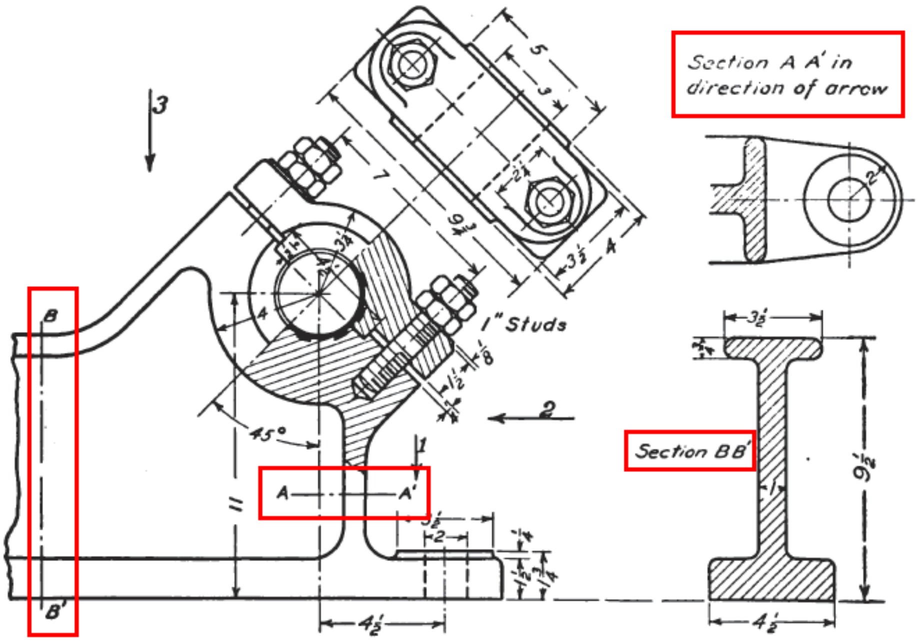
A. section with more planes (parallel or not), in the case of presence of hidden elements that not belong to the same plane;
B. half section, that must follow the convention that imposes to avoid hidden lines in the cross-section plane intersection;
C. more than one cross-section plane in the same drawing, to exploit all the possible views;
D. Partial section, let to interrupt a view using the simple thin line.



A) In this example, a cross-section with parallel planes allowed to better describe the object in a more efficient way, avoiding the use of hidden lines.



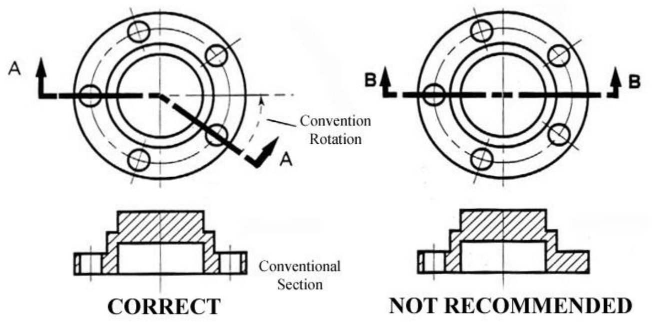
In this case, the concept express in the figure on the left is used again. In this case, some elements are rotate (as indicate) to describe entirely the body in only one view.



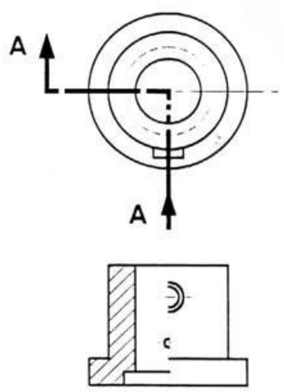
C) More than one cross-section plane is used to completely described the object in all its geometries.

Moreover, it can be seen a partial section (D) that is not indicated with a tick line but, following the convention, with a thin line. This section shows the presence of two different elements, connected using a screw: in this way, the coupling can be completely dimensioned.

Eventually, this is a good example of how to completely describe a complex geometry object with the minimum number of views.



As in this case on the left, a single and straight cross-section plane was not enough to describe all the holes present in the object. For this reason, the cross-section line is rotated and the elements of the body are completely described.



Half section example (B). It can be noticed how no tick line is used to indicate the cross section.

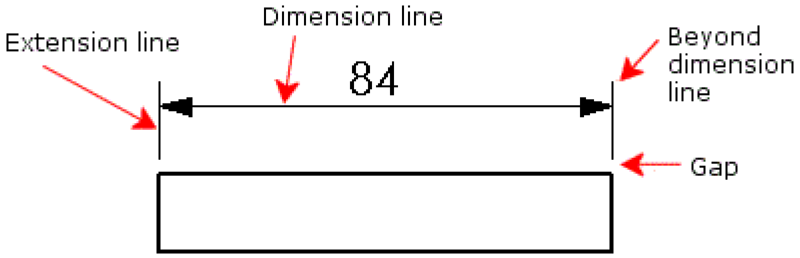
The main purpose using sections is always to add information inside the drawing, so always use the best section to describe completely and clearly the item with the minimum number of view.

Useful links (even for the next chapter):

https://ocw.mit.edu/courses/mechanical-engineering/2-007-design-and-manufacturing-i-spring-2009/related-resources/drawing_and_sketching/#Where_To_Put_Dimensions

DIMENSIONING

In general, if we are dimensioning it means that we are providing all the necessary dimension information to completely describe the object. There are several ways for dimensioning and different procedure that may request some experience (and that in general are not unique).



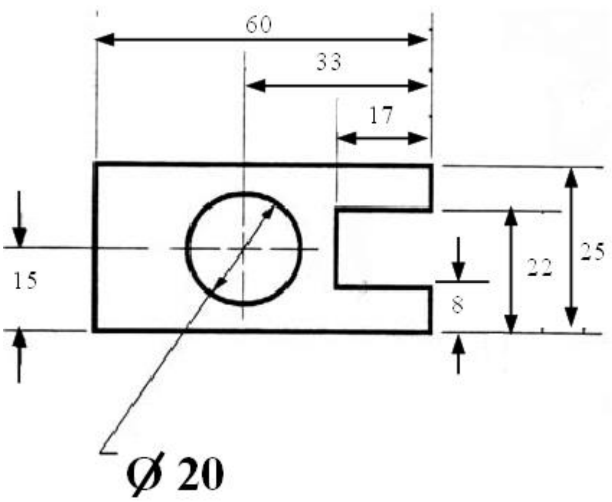
In the figure, we can see the elements needed to dimension and how to insert them in a drawing.

In this case, the number is above the dimension line according to the European standard. Other positions can be accept, depending by the standard.

Here we present some practical rules to dimensioning, in general, in a correct way:

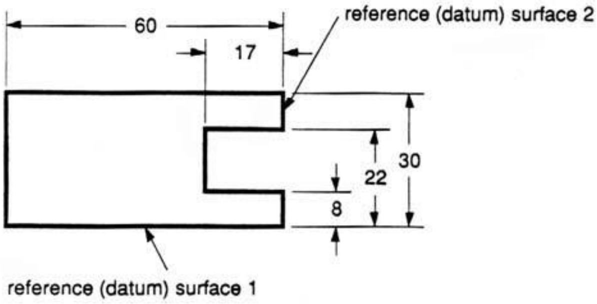
- The dimension line should be **thin**, **B** line, **arrowhead** at each end, connected to the drawing thanks to an **extension line**
- All the dimensions must be in the **same unit**
- Every dimension must be reported in the drawing only one time, even if is present in more than one view, so **avoid redundancy in dimensions**
- It is better to **avoid** dimension that is given by the **sum** of others
- The dimensioning must be **complete**, so put as many dimension as are necessary, no more no less, and no size must require the use of the scale to be read
- Put the dimensions in the **view** which most **clearly describe** the object
- Dimensions of **hidden lines** must be **avoided**
- **Diameters** and **radii** should be specified with the **appropriate symbol (Ø)** preceding the numerical value
- Use a **reference standard** for the dimensions, as a surface or a line (***datum***), for example measuring from one end of a shaft to various points
- Each must be associated with the corresponding **tolerance**, according to the standard.

Here following some examples.



An example of how insert different kind of dimension in the same view. In this case, some vertical measurements (15 and 8) are insert in the middle of the dimension line, using the American standard: this is usually an error. These should be represented as the other two vertical dimensions.

As we can see, the only hole present in this view is dimensioned using the diameter symbol, according to the standards.



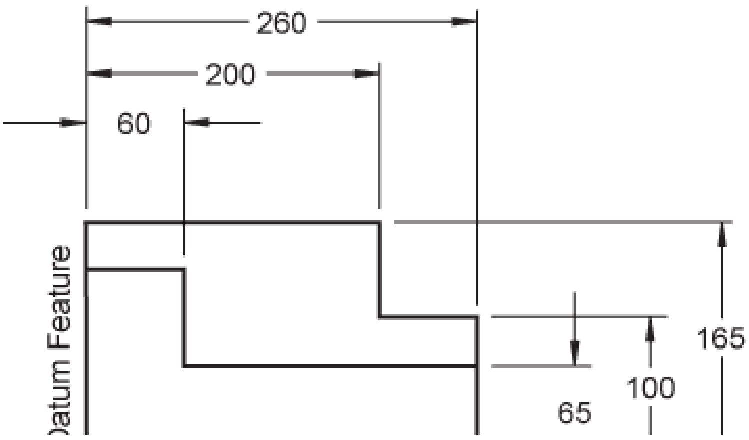
This is an example of how a datum (a reference plane or surface) it can be used to dimension a figure: in this way, all the measurements must be taken starting from the same references.

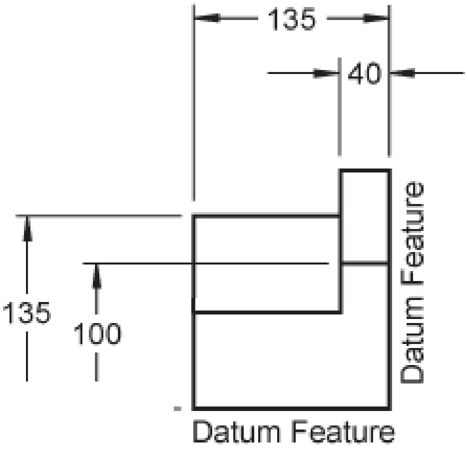
This approach permit to have a clearer drawing, avoiding redundancy and decreasing the error due to the reading of the dimensions.

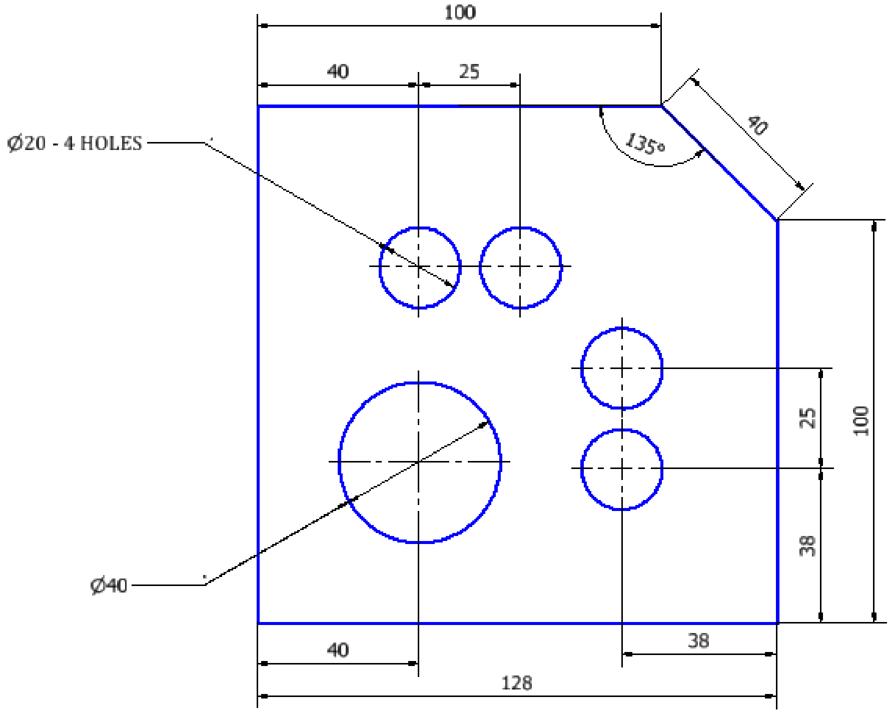
Those two examples (on the left and on the bottom) shows how to use this approach in different view.

REMARK: all the dimensions are insert in the middle of the dimension line using the American standard.

TO CORRECT IT: put the dimensions above the dimension line (see the next example)

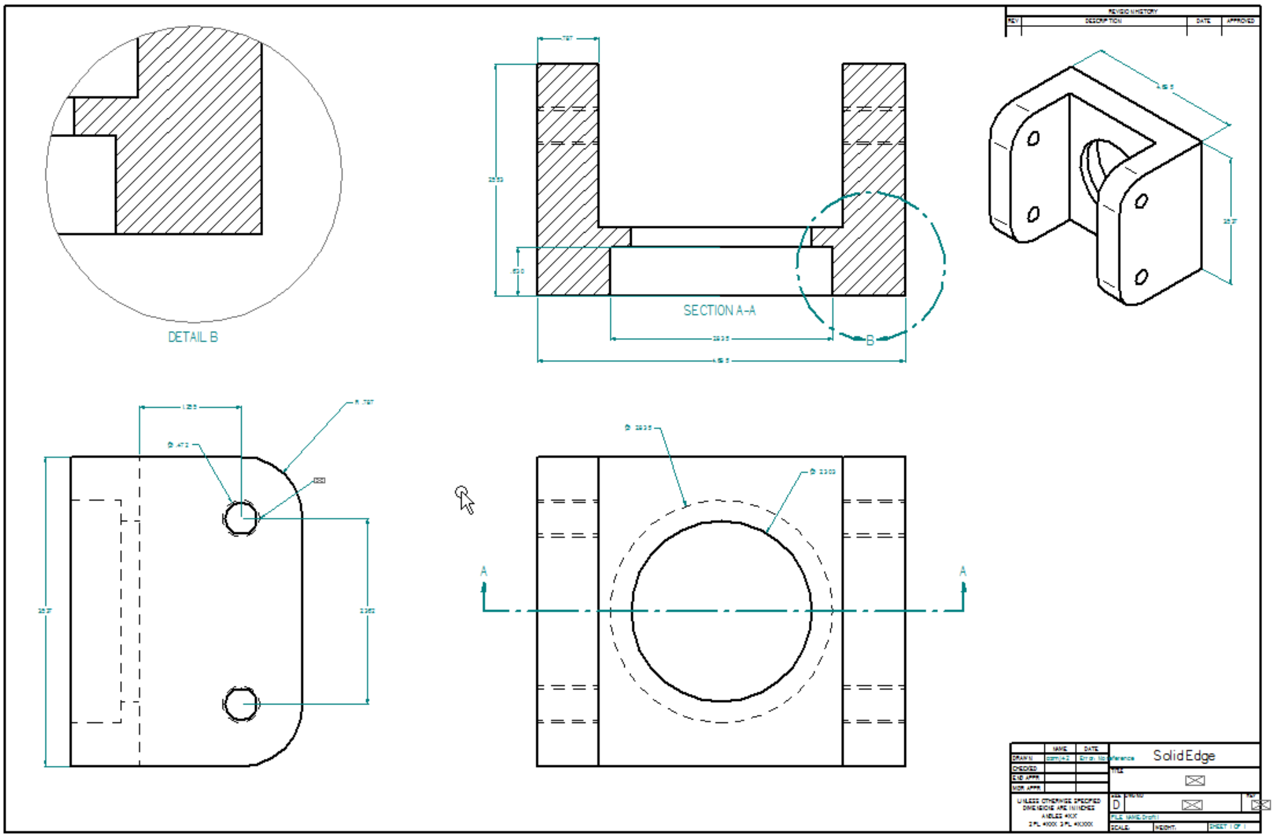




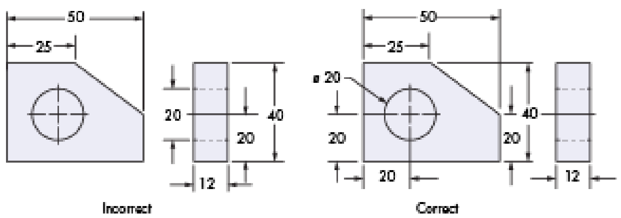


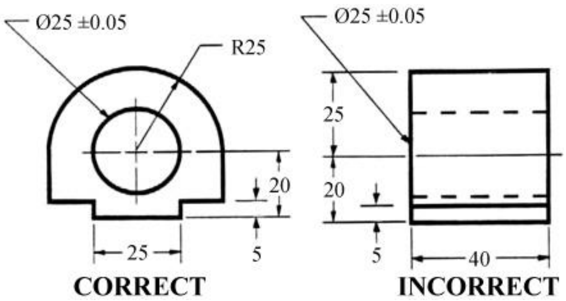
In the example in the left we can see how insert correctly the dimensions above the dimension line. Moreover, it can be noticed how

- the four holes diameter are represented
- all the dimensions necessary to positioning the circles centres are present in a clearly and unique way
- a non-perpendicular angle is represented



Multi-view drawing completely described through the dimensioning. As we said before the dimensioning process is not unique, but this example shows how a section it can be essential to avoid the used of hidden lines, in this case not dimensioned (with few necessary exceptions).

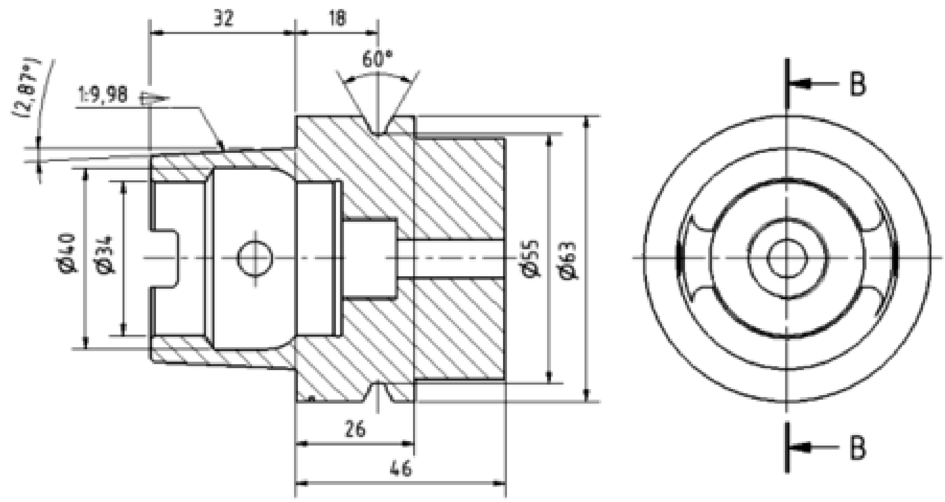




Here two examples representing different errors due to the representation of the dimensions in the wrong view.

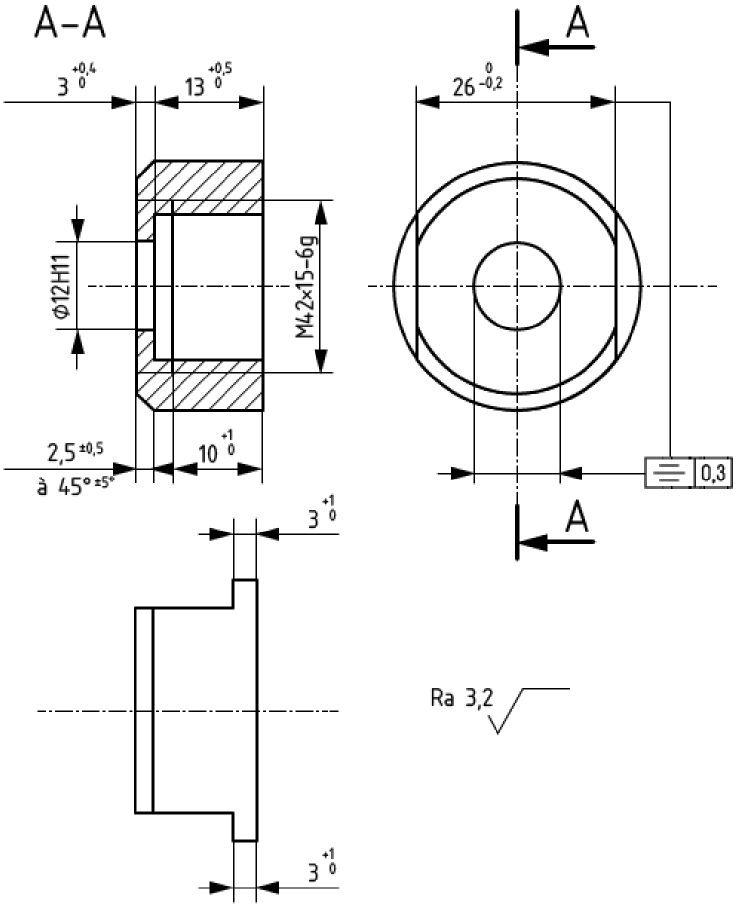
In both these cases the diameter were initially dimensioned using the hidden lines and their centres were not unique define.

Moreover, in the upper figure, tolerances are added on the diameter dimension. These are necessary to coupling the component with another.



The upper example shows how a section is necessary to completely describe a complex geometry of an element that present different holes. In this case, it can be noticed that:

- all the measurements are insert using the European standard (above the line for the horizontal, on the left for the vertical)
- the diameters of the cylindrical elements present the corresponding symbols
- the slope surface presents the necessary symbol and representation (1:9,98)
- the inclination of the central circular hole is indicated (60°)
- the section is correctly indicated on the right view(B-B)



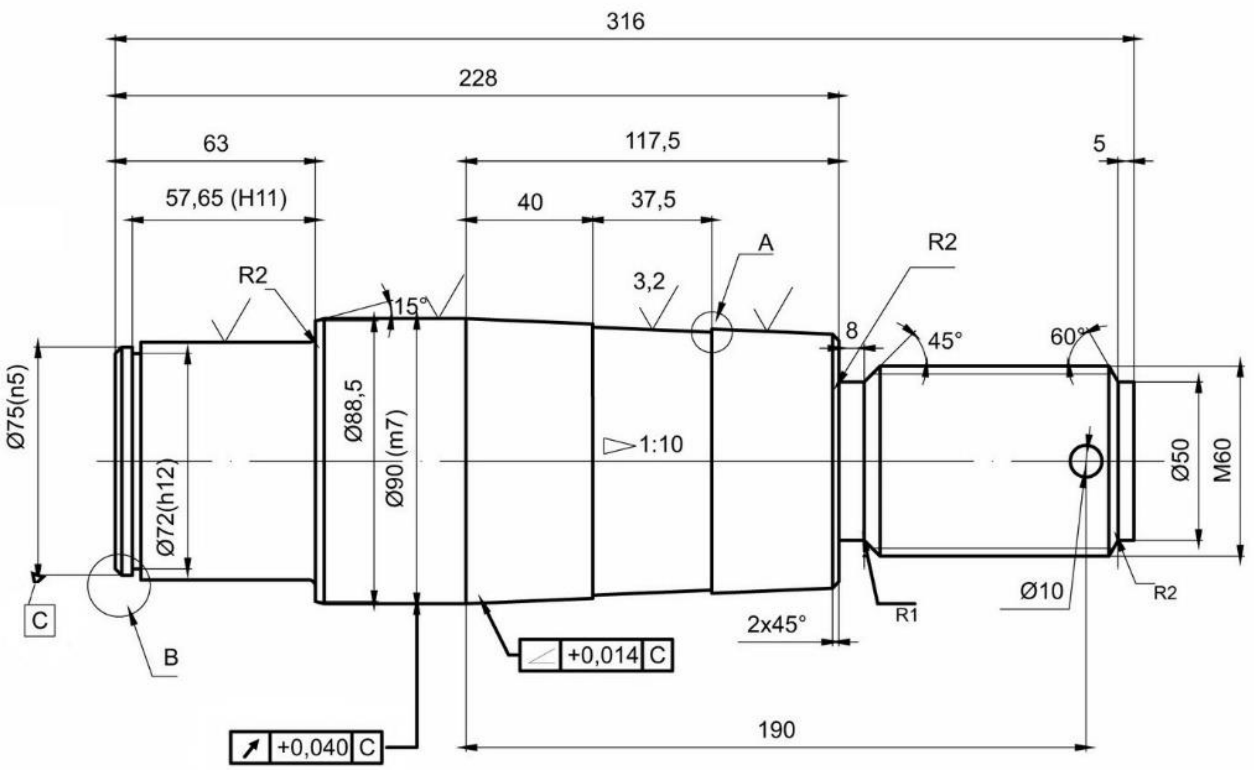
On the left another example of a section completely dimensioned.

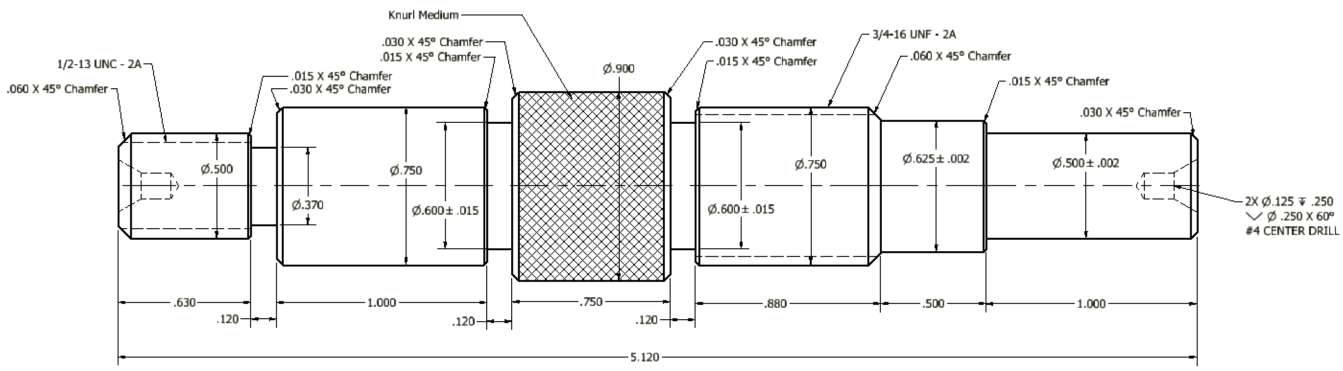
It can be noticed the representation of chamfer of 45° on the left view.

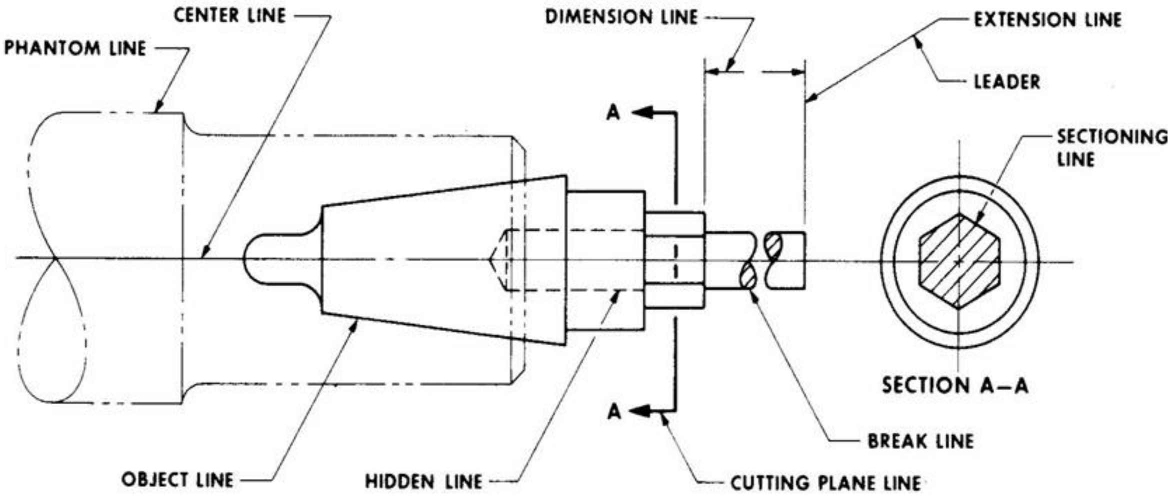
Here all the geometrical tolerances are represented, according with the standards.

EXAMPLES

Here following more complex examples of how to completely dimensioning shafts.







A practical resume of all the lines that can be present in a drawing.

Useful links: For more detail about the subject, we suggest the following links:

http://engineering.pages.tcnj.edu/files/2012/02/dimensioning_and_tolerancing.pdf

https://ocw.mit.edu/courses/mechanical-engineering/2-007-design-and-manufacturing-i-spring-2009/related-resources/drawing_and_sketching/

And here a guide to common errors:

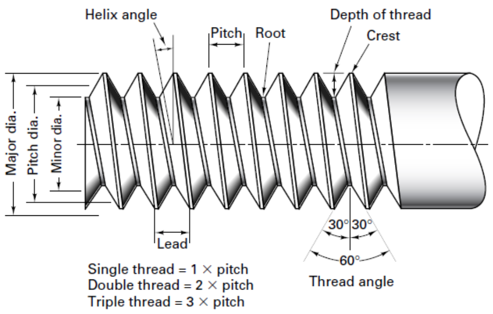
http://machinedesign.com/cad/8-common-drawing-mistakes-avoid

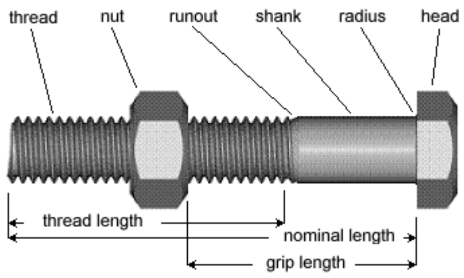
http://www.epicness.us/textbook/files/index.htm

http://www.epicness.us/textbook/files/dim/dim_page3_ex5.htm

THREAD AND SCREWS

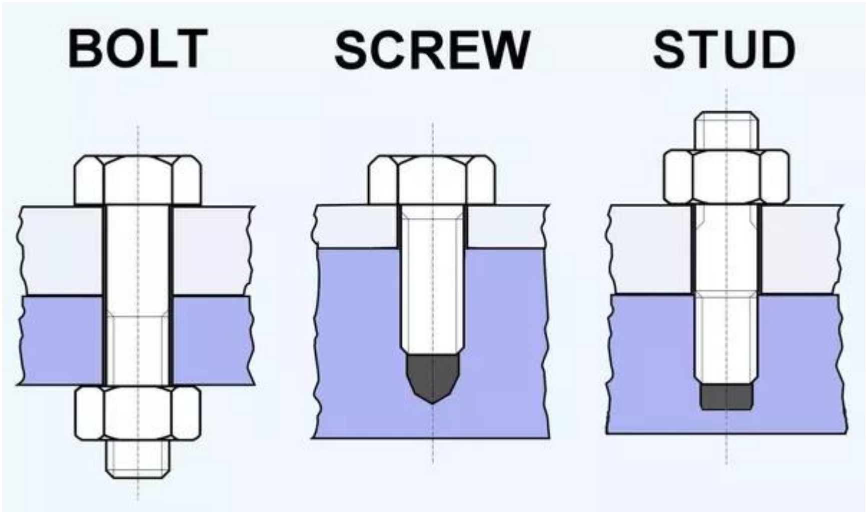
The ISO metric thread profile, represented in the figure below on the left with the characteristic nomenclature, is the basis to realize a joint using a screw.





To realize a non-permanent joining, screws can be used in several ways:

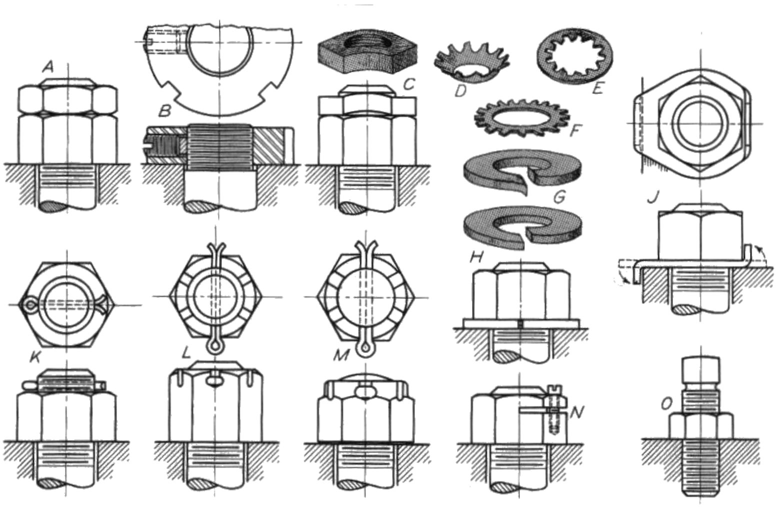
1. **Bolts and nuts**, used when both the sides of the component are accessible. In this case, the hole inside the two components does not present any thread. In some conditions, the used of locking devices, as spring washer or lock nuts, are required to avoid the unscrewed of the joint due to vibrations.
2. **Set screws** are used when only one side is accessible. In this case, only the last component present a thread that realizes the actual joint with the screw.
3. **Studs** are used when is not possible clamp the screw directly inside the component due to the composition of the material, that could lead to damage to the thread. In these cases, the tightening is realized by using a nut. These kinds of screw are used when heavy pressures are present in the joint.



The are many different kinds of **locking device**, but the most common are:

- **Self-locking nuts** (figure on the right);
- **Spring washers**, of different shapes (D, E, and F in the figure below);
- **Jam nut** (A in the figure).

In the following figure, some examples of locking devices.



REMARK:

- A screw **cannot work** with a **shear** stress
- **Never thread both** the component that must be joined, otherwise a shear stress is applied to the screw.

Useful links: For more detail about the subject, we suggest the following links:

http://www.staff.city.ac.uk/~ra600/ME1105/Lectures/ME1110-16.pdf

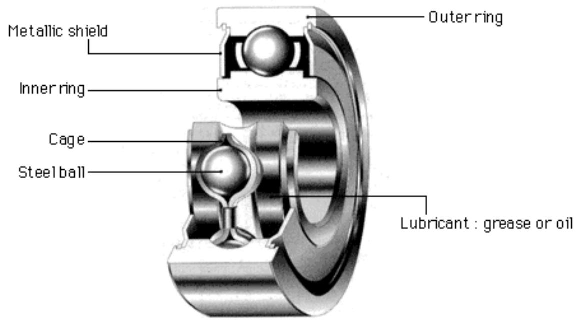
1. locking devices and general review:

http://www.me.metu.edu.tr/courses/me114/Lectures/loc_key_spr_riv.htm

DRAWING: SECOND PART

In the second part of this paper some more complex topics related to machine design will be exposed, even if some of them will be part of the program of the course of *Applied Mechanics*, to provide a complete overview of all the fundamental arguments related to the mechanical drawing.

BEARINGS



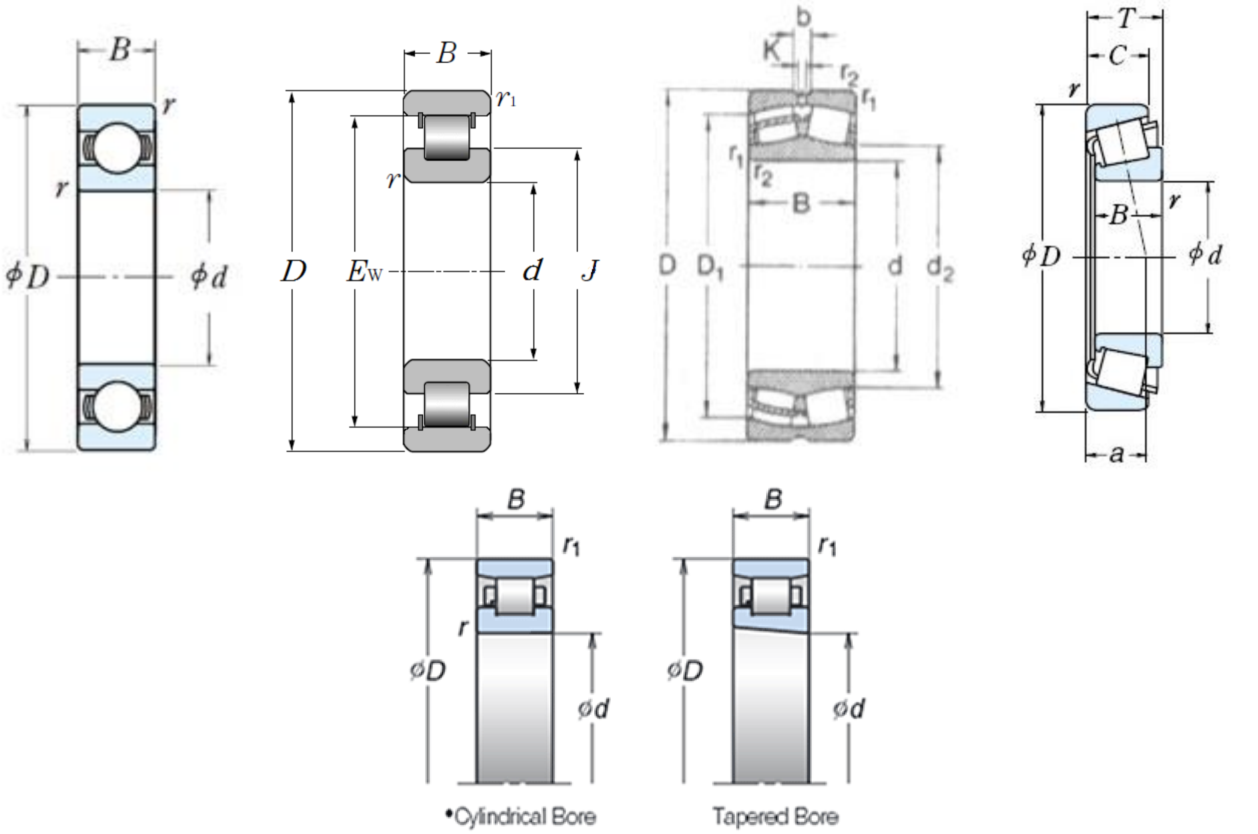
The bearings are the support elements that allow the relative rotation of one body respect to another, reducing the friction effect between moving parts, thanks to the rotating elements (balls or rollers) inside of them.

There are different types of bearings: they are chosen for a specific application depending on the load direction, the type of operation ant the motion allowed.

We can classify the bearings depending on the rotating elements as

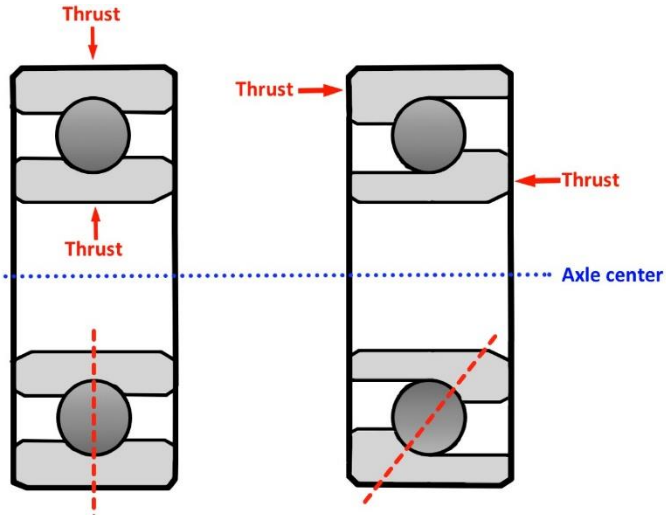
a) *Ball Bearings*
b) *Roller Bearings*
    1. *Spherical roller bearings*
    2. *Tapered roller bearings*

In the same order in the following figure



Or we can classify them depending on the allowed load direction as

a) *Radial*
b) *Axial*
c) *Oblique*
d) *Orientable*
e) *Single effect/double effect*

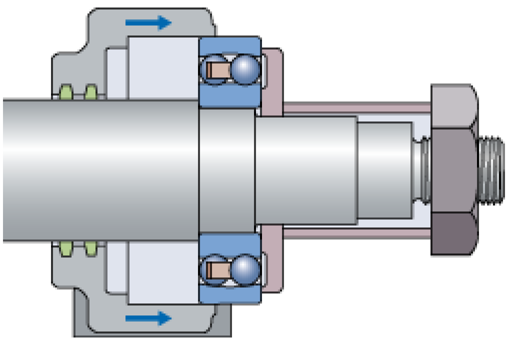


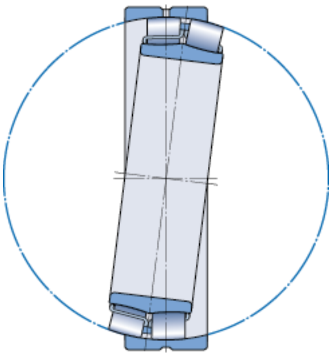
In the figure a Radial and Oblique (from left to right) are present.

As we can see, the two red-dashed lines represent the load direction.

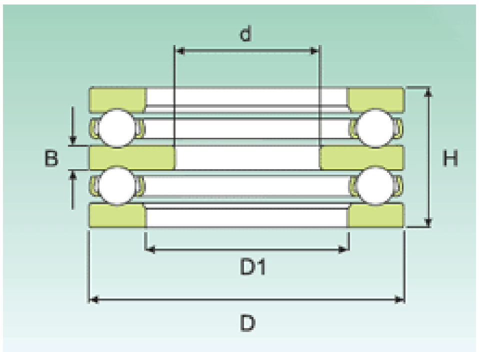
To keep the bearings fixed some elements must to thrust in the red direction, as the arrows in the figure show.

The previous concept is underline in the figure on the right, where we can see an element to avoid any movement of the bearing, due to the force apply on it (blue arrow).



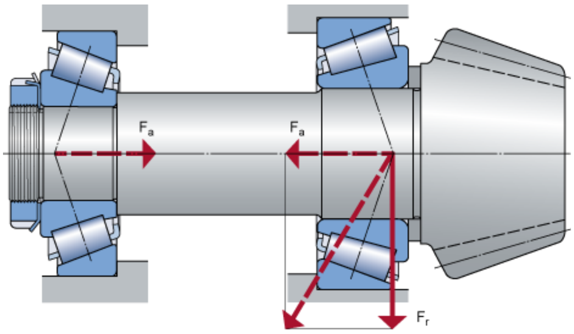


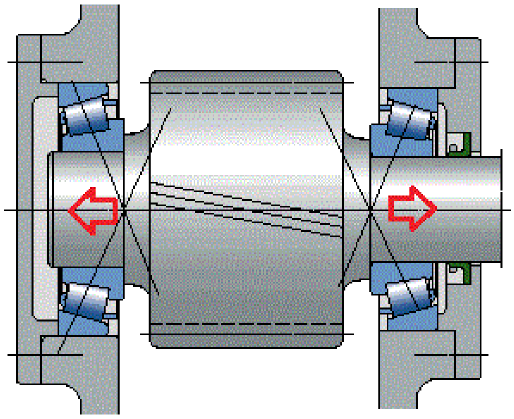
An orientable bearing (on the left) and a double effect bearing (on the right) are represented. As before, the direction of the load defines the bearing types and determines where the elements to fix this component must be insert.



Then we can install them using several configurations and ways

- *O*
- *X*
- *Axial*
- *With Springs*

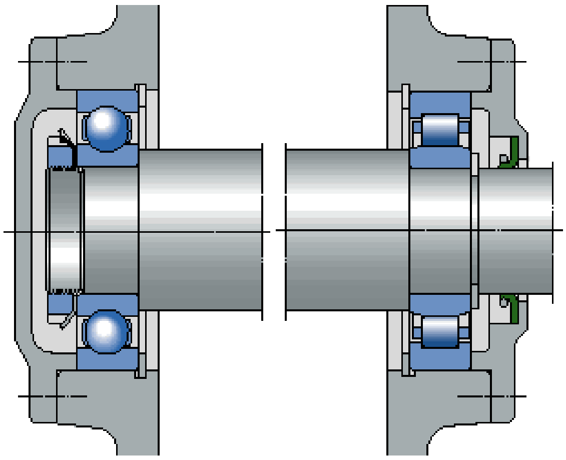




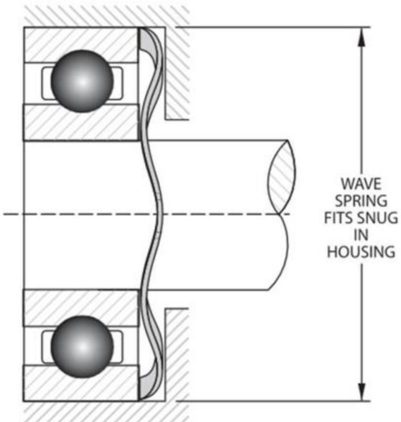
Here an O configuration (on the left) and an X configuration (on the right): those names comes from the relative radial axis shape (in black on the left picture).

As we can see the two configurations allowed opposite axial load (in fact, the resultant force is always on the same direction of the perpendicular axis of the rotating element) and are used when:

- Low bending moment and **axial load** → **X** configuration;
- High **bending** moment → **O** configuration;

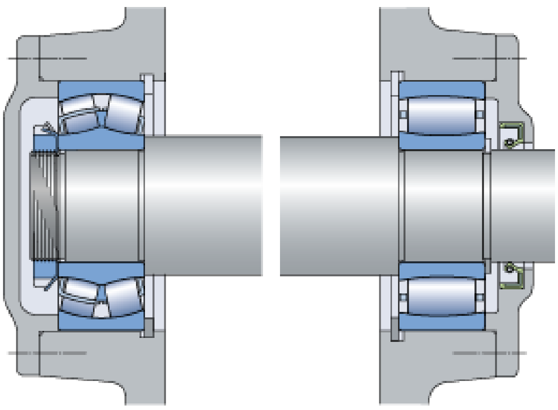


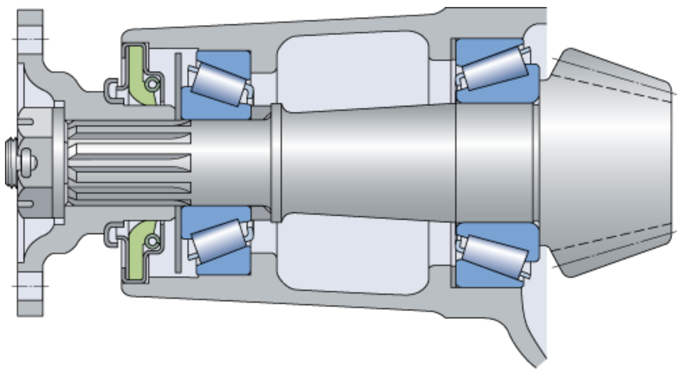
In this case the two bearings are both axial and fixed in different ways: on the left using a thread and using an elastic ring on the right.



In the above figure, we can see how a preload spring is used to mount the bearing, to provide advantages cases.

Here following some other examples, respectively X configuration with an axial bearing on the left and O configuration on the right figure.



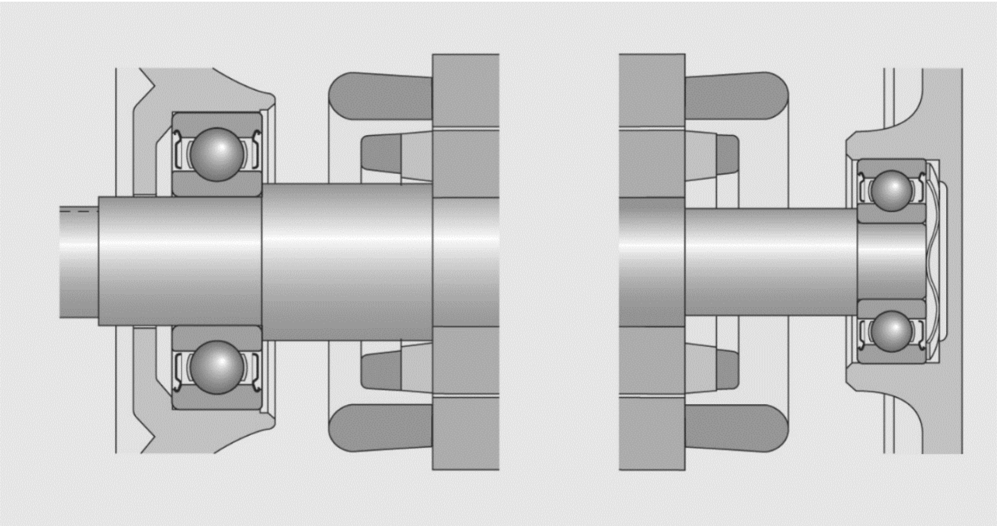


REMARK: Few important practical **tips** are here reported to help the reader to choose in a correct way how to **mount a bearing**:

1. The **axial** degree of freedom must be **locked**;
2. Where it is possible, **only one axial support** of a shaft must be locked to allows axial expansion of the component (for example, due to thermal effect);
3. Chose the configuration of the bearings **depending on the load**
   - **Axial load** (with low bending moment) → **X configuration;**
   - **Bending moment → O configuration;**
4. Sometimes it is necessary to add elements that applied to the supports a **preload**, in order to **avoid** the "**skidding**" (loss of contact between the sphere and the cage).

The following picture shows an example where a preload is applied to the right bearing using a spring. In this way, the contact between the spheres and the cage of the bearing is always guaranteed, even if the shafts is expanding in the axial direction due to temperature effects.

As it can be noticed, only one side of the shaft (in this case on the left) is completely lock in the axial direction, according to the tips number 1 listed before, instead, the other part (thanks to the spring) allowed the shaft to have small displacements in the axial direction.



Useful links: For more information about bearings we suggest the link of one of the main manufacturer of this product:

http://www.skf.com/uk/index.html?switch=y

SPRINGS

The springs are elastic elements that can accumulate elastic energy coming from an external imposed deformation.

They are commonly used to:

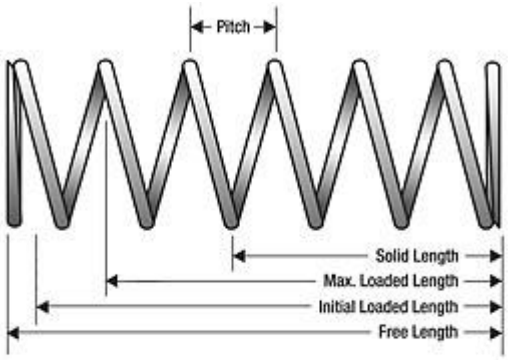
- Leave the relative deformation between elements without compromise the contact point
- Bound the effect of impact and deformation, converting the kinematic energy in potential energy
- Bound the vibration transmission between different components

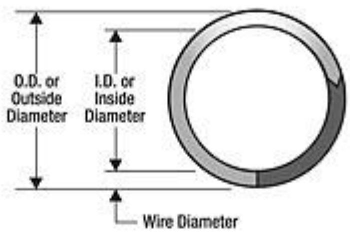
The characteristic of a spring is given by the trend Force/Deformation and by the stiffness k.

The last one is typical of a spring and depend on the shape, the material and by the configuration of the system.

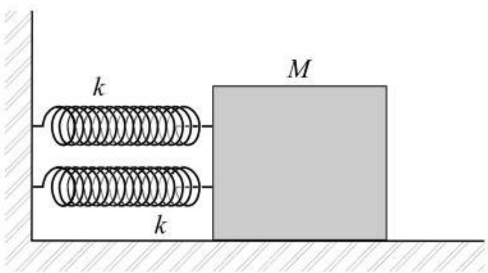
To design a spring, it is necessary to set:

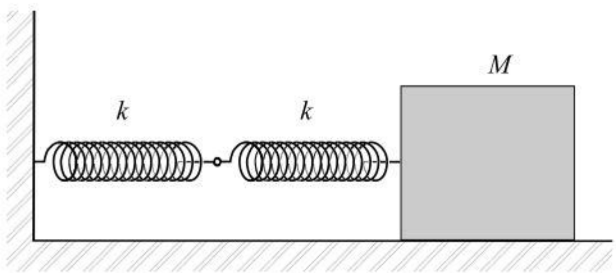
- Pitch of the spring (p)
- Number of effective coils (i)
- Total number of coils (n)
- Diameter of wire (d)





More than one spring can be put inside a system in *parallel* or in *series*. In the first case the equivalent stiffness k_{eq} is given by the sum of the two stiffness, in the second case, the inverse of the





PARALLEL (total displacement: x = x1 = x2)

F = F1 + F2 = (k1 + k2) x

SERIES (total displacement: x = x1 + x2)

F = F1 = F2

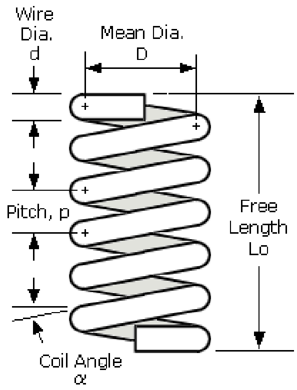
x = F/keq = F/k1 + F/k2 = (1/k1 + 1/k2)

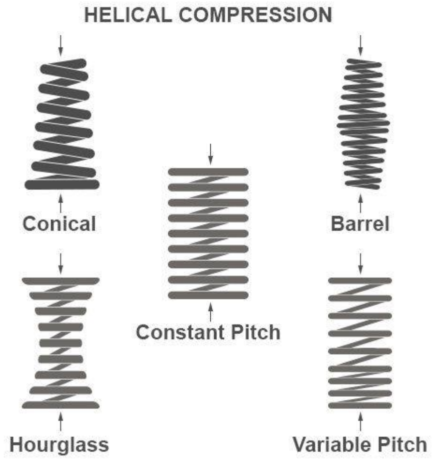
equivalent stiffness is given by the sum of the two inverse.

The classification of the springs is given by the shape, by the external excitation or even by the deformation induced by an external load: all these can define the same spring. The most common are:

- ***HELICAL SPRINGS (CYLINDRICAL)***

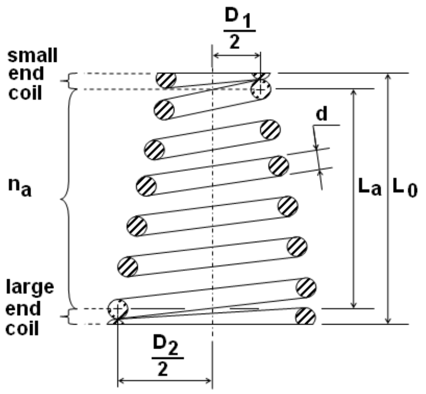
They can be under compression or traction, depending on the load along the spring axis. The deformation is given by torsion.

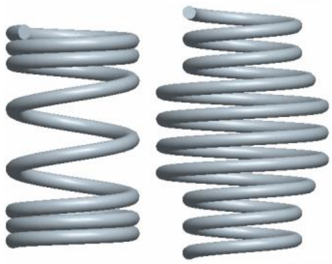


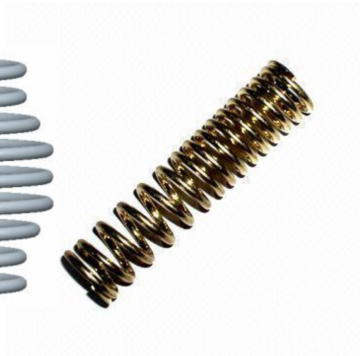


- ***CONICAL HELICAL SPRINGS AND WITH VARIABLE PITCH***

These kinds of springs give a nonlinear characteristic.

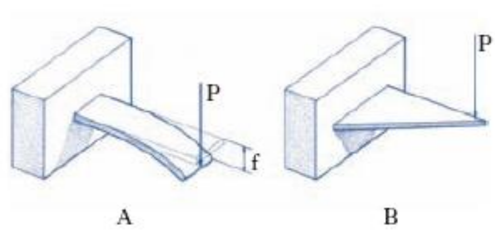






- ***BENDING SPRINGS (RECTANGULAR, TRIANGULAR AND TRAPEZOIDAL)***

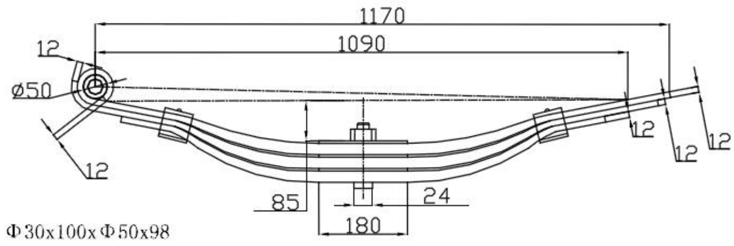
Those springs are used when the elastic element must work in a limited space. They are subject to flexural deformation, different by the geometry. These leads to the next kind of springs.



- ***LEAF SPRING***

This spring is equivalent to two trapezoidal springs with the same base: it is like a pack of leaf given by the section of two trapezoidal springs. The Main differences are:

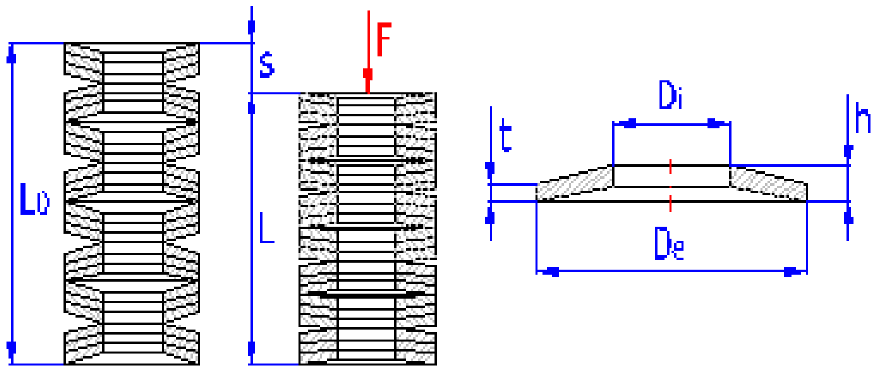
- They need a lubricant due to the friction between the different leaf, that dissipate energy;
- The displacement, in this case, is not in the same plane of the original trapezoidal spring.



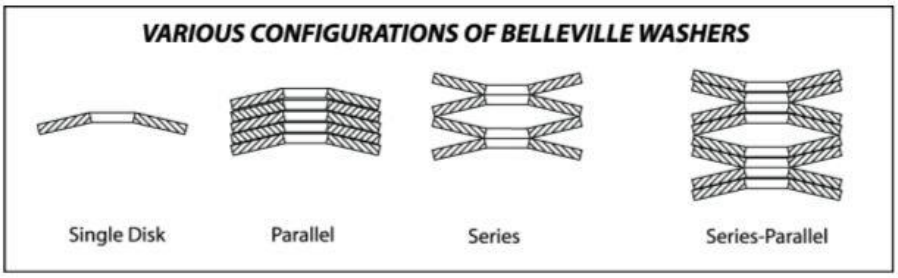
- ***Belleville spring***

The stiffness of this kind of spring change with the displacement

Usually, this kind of spring is used with a number or elements or in a stack, due to the high stiffness of the single Belleville spring. Changing the number of springs and their configuration it is possible to tune the global characteristic as desired.

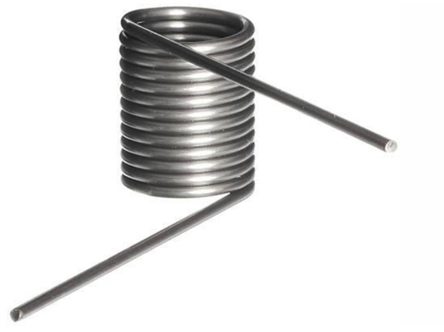


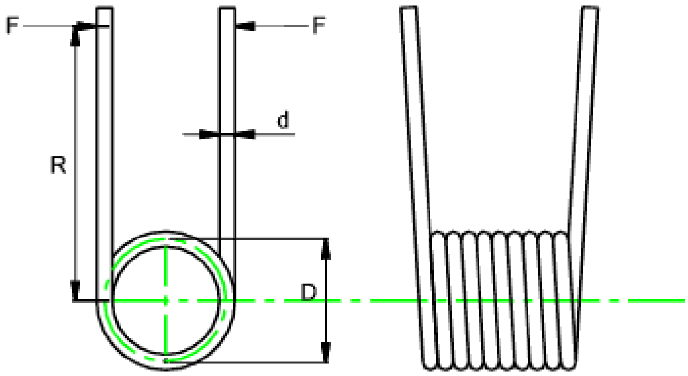
As we can see in the following example, it can be used a single configuration, a series or a parallel configuration or a mix of the two.



- ***Torsion spring***

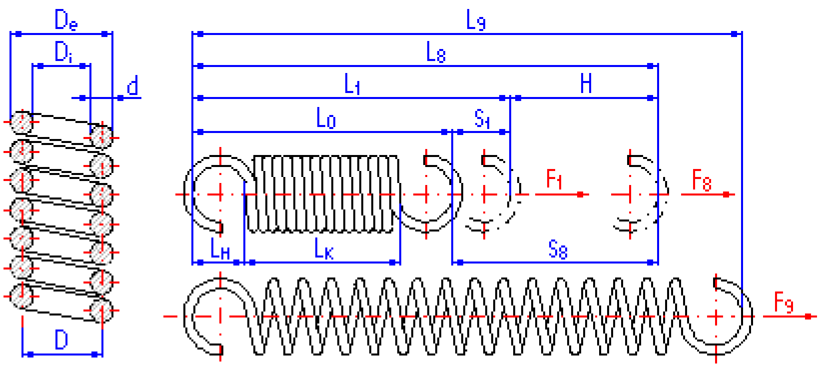
These springs have a flexural displacement. Moreover, they have a pivot inside to avoid lateral flexion during the assembly process.

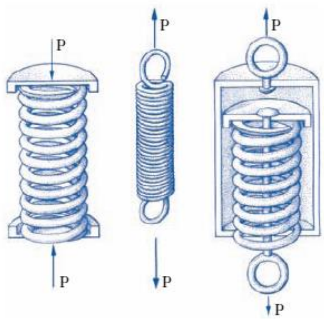




- ***Extension spring***

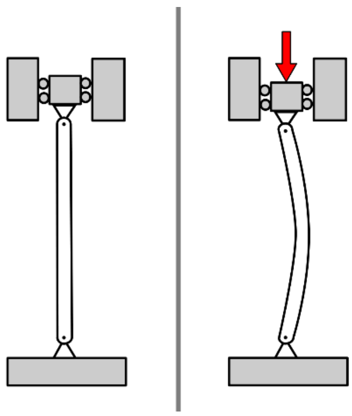
This spring is given by different rings assembly together: internal are cylindrical and external are conical (or vice versa). This kind of spring present a sort of hysteresis characteristic and are used in the presence of an impulsive force.





Buckling

When the deflection of slender compression of a spring exceeds some critical value the spring is in a critical phenomenon called *buckling*. In this case, the spring became unstable, as it can be seen in the figure on the right.



GEARS

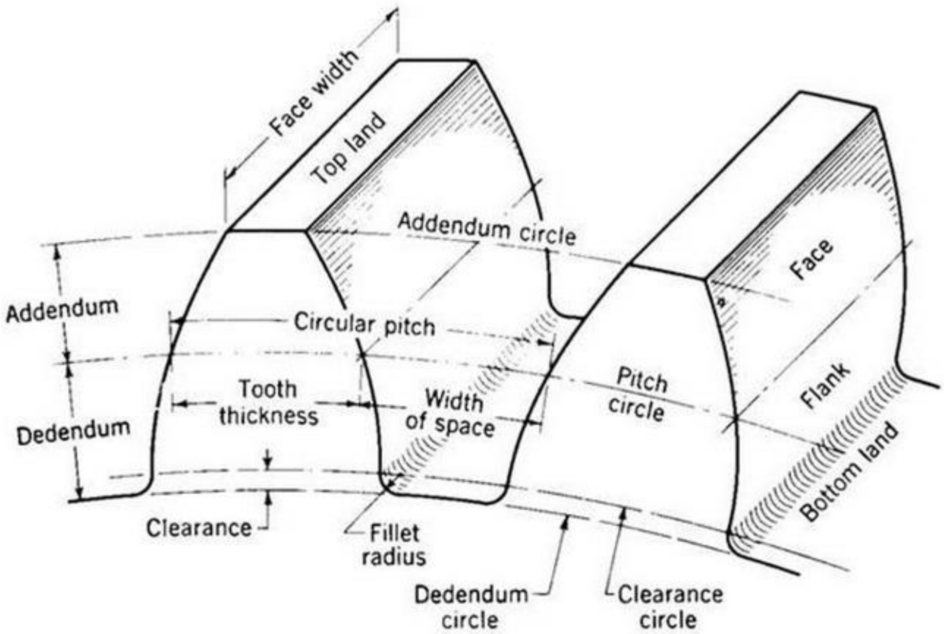
A gear is a toothed wheel which is used to transmit power and motion between machine parts, using the mesh of the teeth of the different elements.

The gears could have different shapes, dimensions, the number of teeth (that characterized the coupling) and other geometric characteristics.

A gear train is a combination of two or more gears to change the speed or the direction of motion of a system.

NOMENCLATURE

Here some nomenclature related to the gears, important to understand the geometric characteristic of this kind of component.

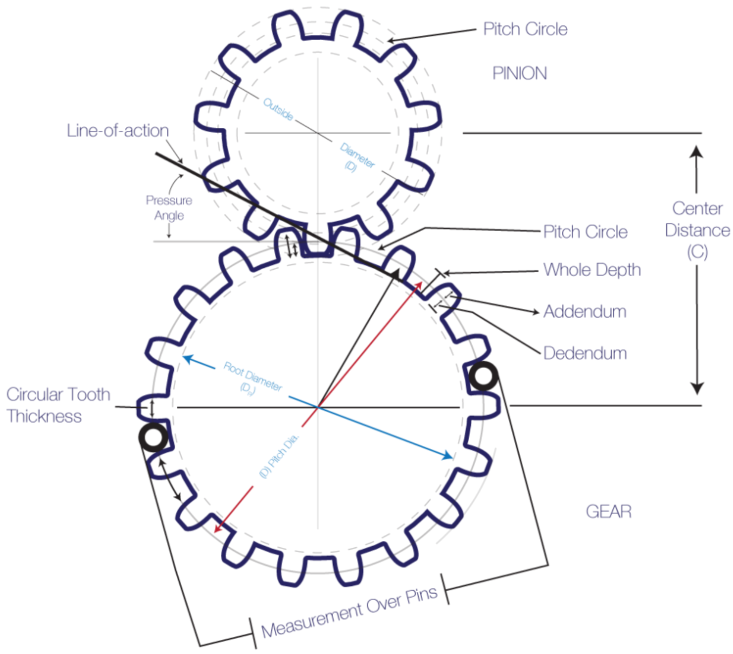


- **Pitch:** Point where the line of action crosses a line joining the two gear axes
- **Pitch circle**: imaginary circle specifying addendum and dedendum
- **Circular Pitch:** Distance of pitch circle from a point of one tooth to the corresponding point on the adjacent tooth
- **Root diameter:** diameter of root circle
- **Addendum**: Radial distance from pitch to top of tooth
- **Dedendum**: Radial distance from pitch to bottom of tooth

- **Outside diameter:** diameter of addendum (outside) circle

IMPORTANT THINGS:

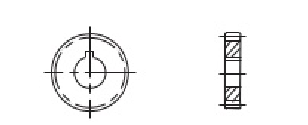
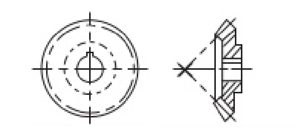
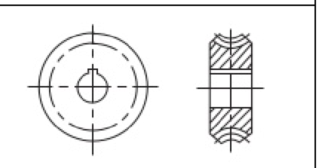
- Two gears mesh each other at the pitch point
- Line of action (tangent to the base circle of pinion and gear) and pressure angle (angle between the tangent to the pitch and the line of action)
- Diameter measurements of the gears
- Center distance C



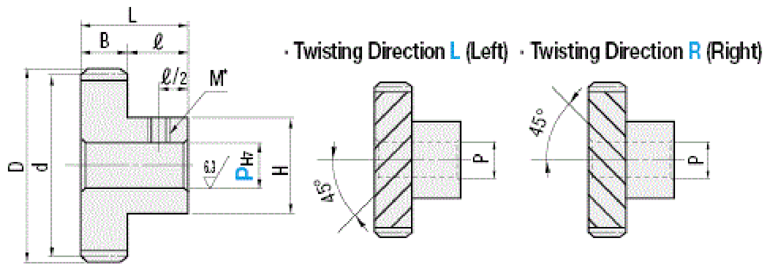
HOW TO REPRESENT IT

- **Frontally,** the teeth are not represented, but only pitch circle and base cylinder (if present)
- **Sectioning**, teeth are not sectioned and are considered straight
- **Lateral** views present symbols (according to the UNI standards) to represent the type of gear teeth.

Here following some examples, according to these three simple rules.

Type	Convention
Spur Gear	
Helical Wheel	
Worm Wheel	

Different gears with their frontal and sectioned representation: in particular, we can see the base circle and as the tooth are represented as straight



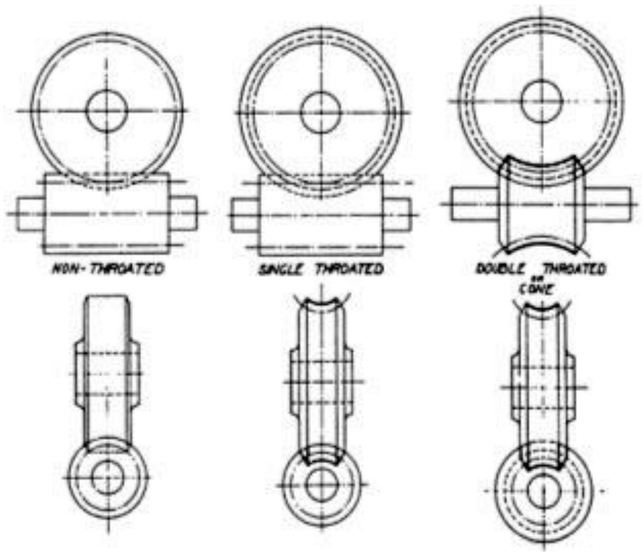
Lateral view of three kind of tooth: (from left to right)

Straight, helical (left) and helical (right).

Sectioning a transmission or a mesh between two gears must keep attention to the following practical rules

- If the two gears share the same plane, they do not cover the contact area of the other
- Sectioning, one tooth overlaps the one of the other gear
- With conical gears, it is better to represent the base conic

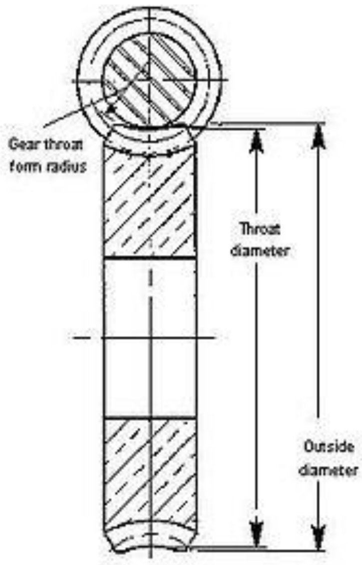
The following examples underline those simple concepts and add some examples of how to dimension a gear.

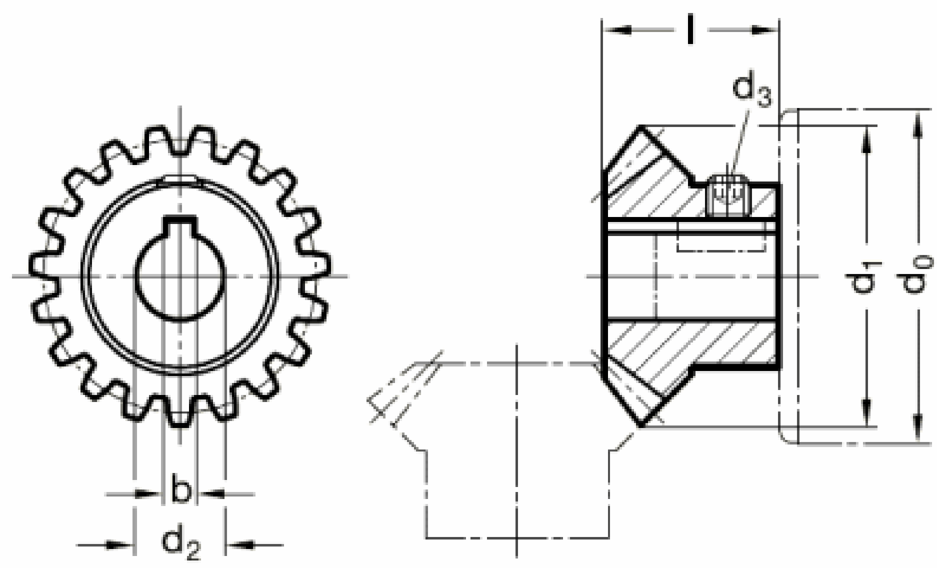


In those examples, in the sections one teeth overlap the teeth of the other gear.

This concept is even more evident in the picture on the right, where the overlapping of the tooth is more clear to the present of the diameters dimensions.

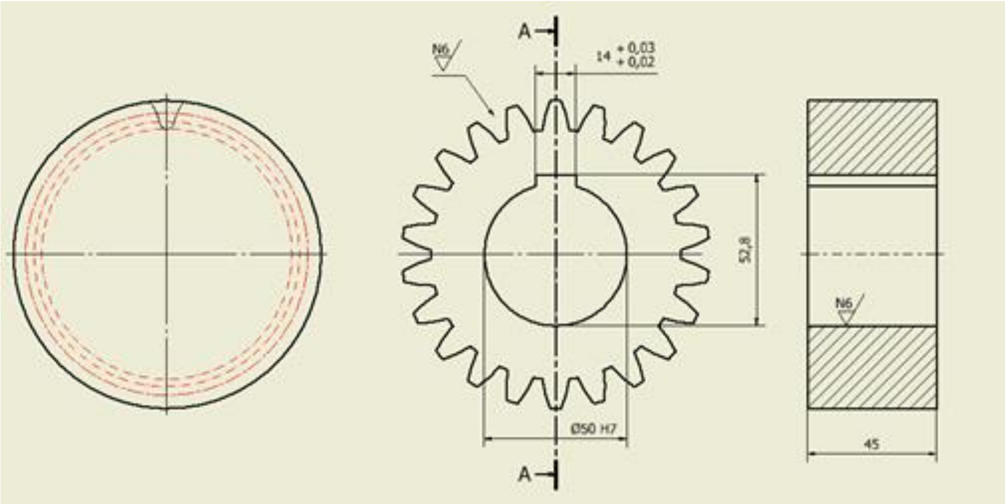
In the last figure on the left we can underline the presence of both the base circles: as we said, this is common to indicate toothed components.



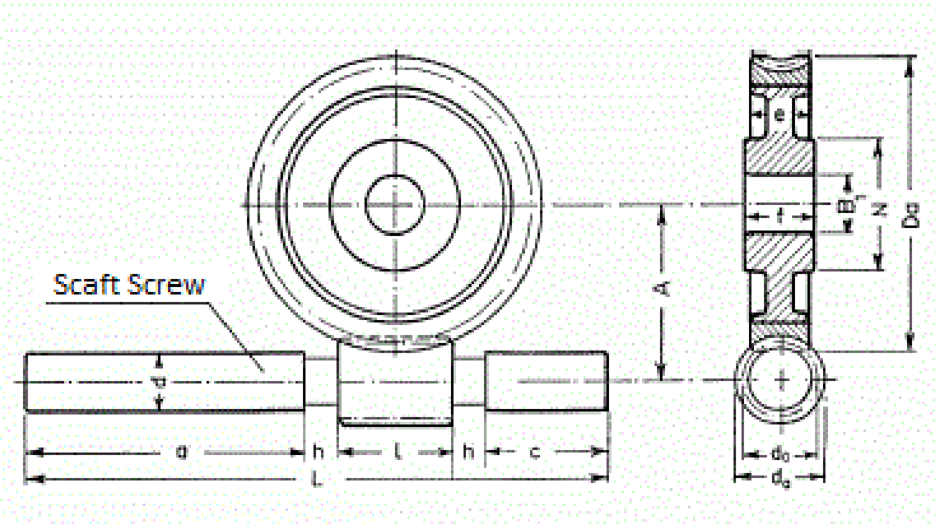


In this picture, it is represented a conical gear: to clarify the shape of the component the base conic is represented on the right.

The dimensioning of the drawing was made according to the rules presented previously.



In the frontal view of the gear it is present only one teeth, according to the convention that simplify the representation of a gear in the presence of a tang: they are both represented with the common axe. This is even an example of how it is possible to insert dimensions and tolerances on a gear draw.



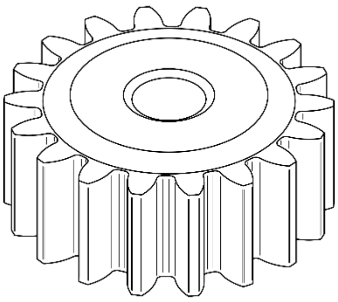
1In this picture we can underline different important concepts:

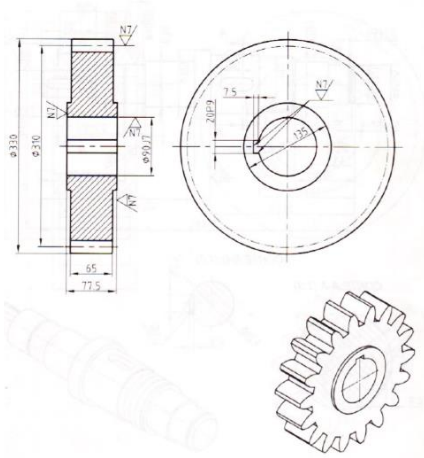
1) The dimensioning is complete and clear, to perfectly define all the elements present in the draw:
    - Two views are used (the right one is a section of the gear), to have a clearer presentation of the geometry of the elements;
    - The base circle and the external one are dimensioned on the section view;
    - the dimension A is important to the mesh of the gear with the shaft (without it the two components cannot be mount together);
2) The gear and the shaft (in the middle part of length l) present dashed thin lines, to indicate the presence of tooth on the two elements;
3) The section in the right present the overlapping of the tooth of the two components, exactly as is indicate in the previous chapter;

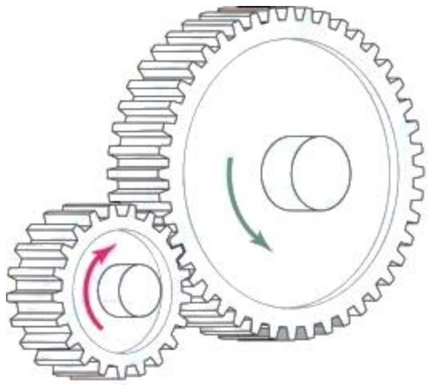
TYPES

There are many typologies of gears, depending on the shape of the teeth and the meshing of the gears involved in the transmission. Here we can see some examples:

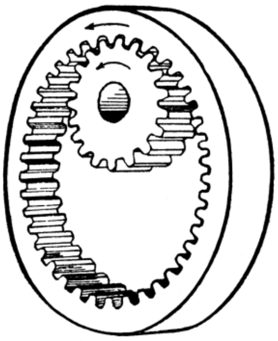
- ***SPUR GEARS***



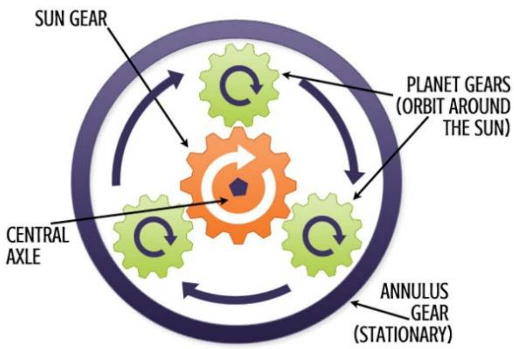


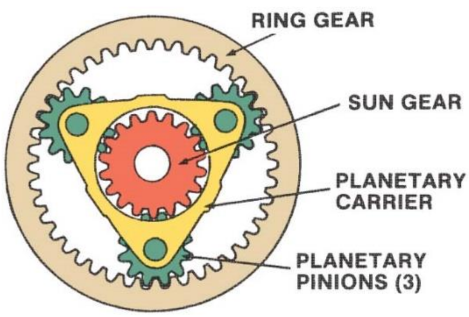


- ***INTERNAL GEAR***

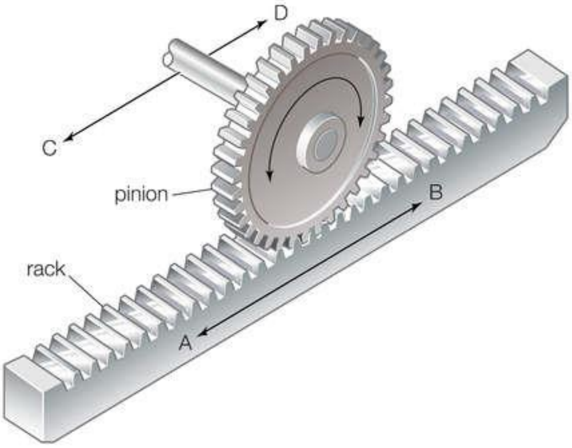


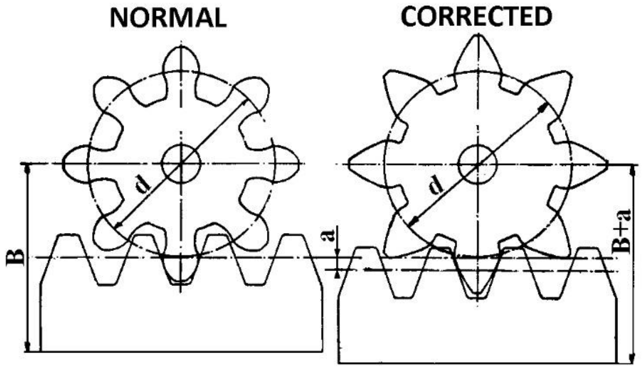
- ***PLANETARY***



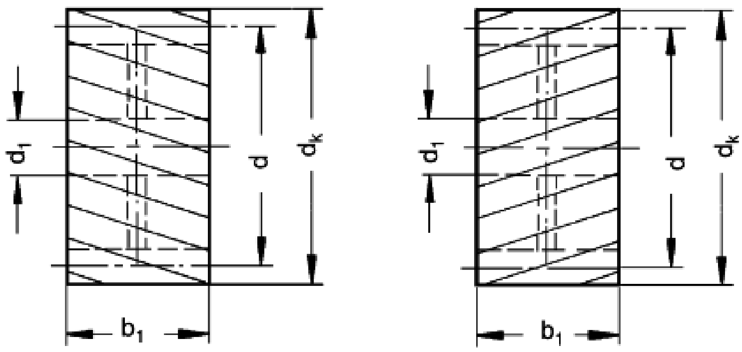


- ***RACK AND PINION***



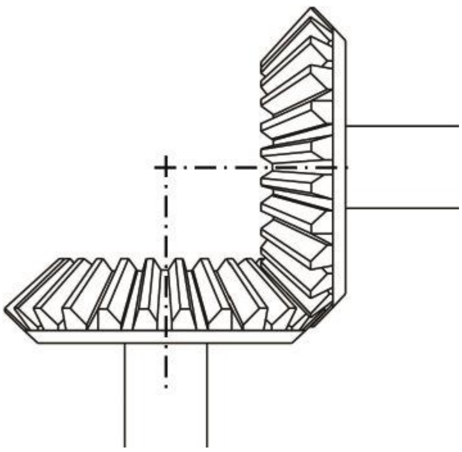


- ***HELICAL***

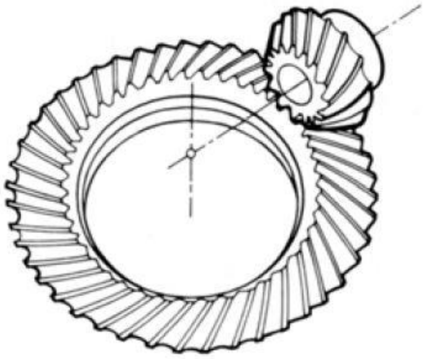


Right hand and left hand helical gears representation

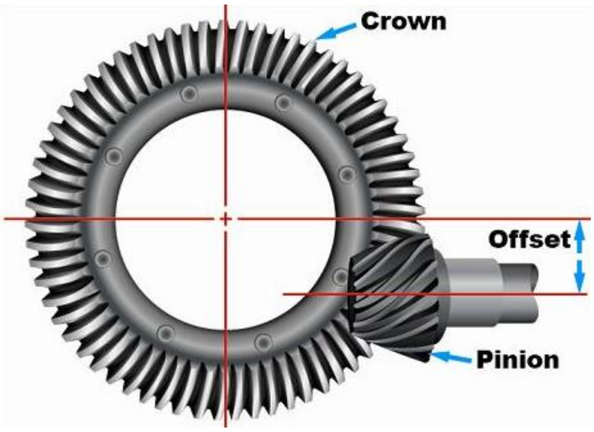
- ***STRAIGHT BEVEL***



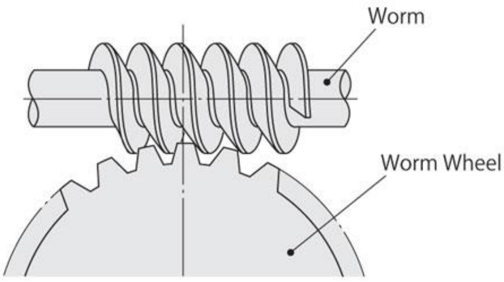
- ***Spiral bevel***

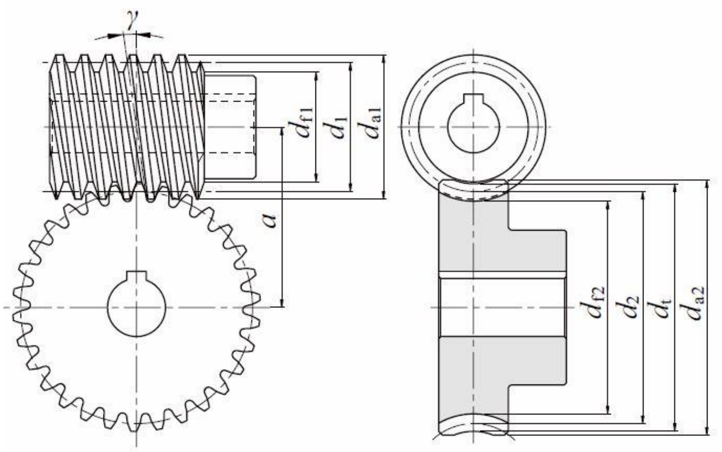


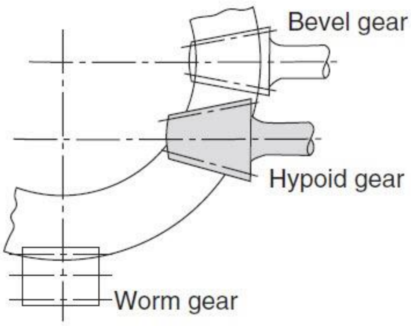
- ***Hypoid gears, transmit the movement between two offset shafts***



- ***Worm and gear***

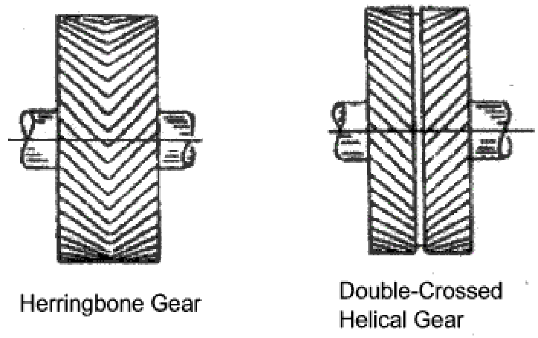






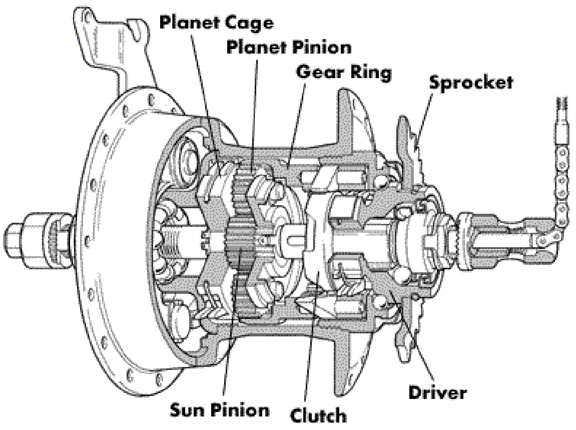
Here we can underline the differences between that three kind of gears, depending on the shafts axis position.

- ***HERRINGBONE***

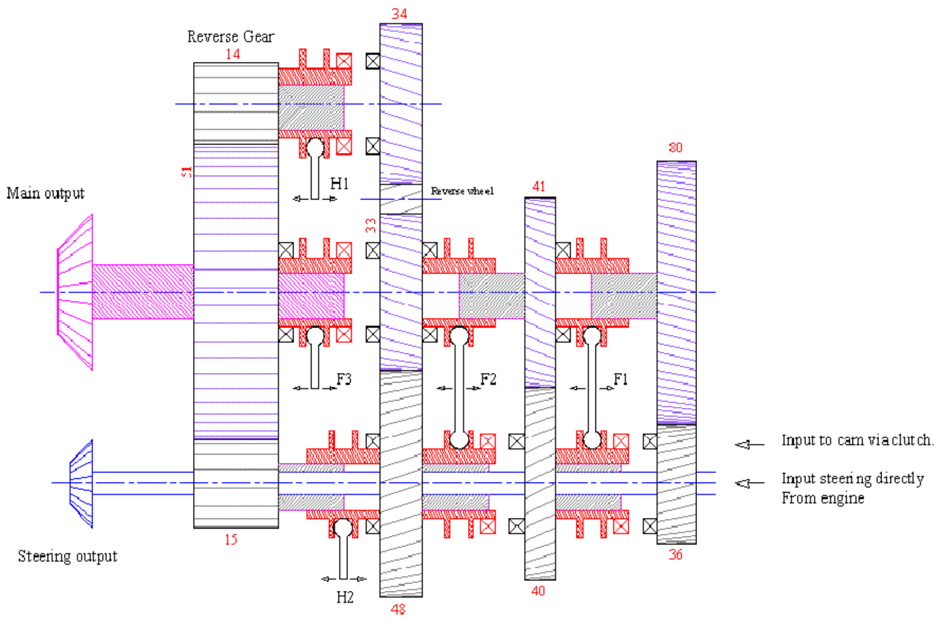


Main difference between helical and Herringbone gears

<u>EXAMPLES:</u> different kind of gears are present:



Partial section of a planetary gear, connected to a driveline using a clutch

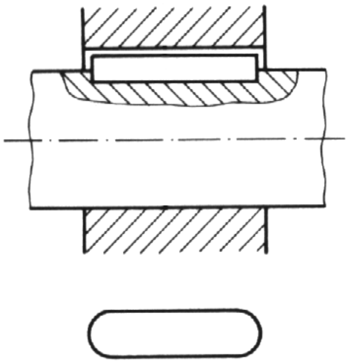


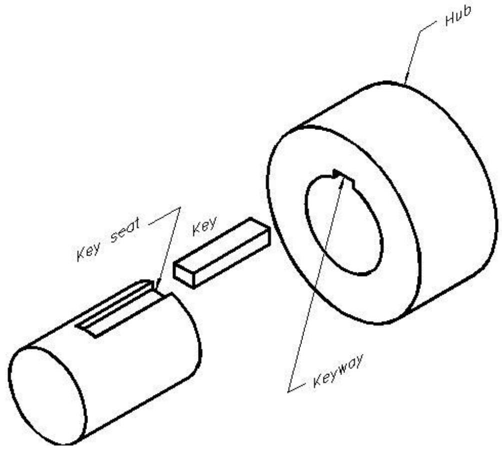
Example of a gearbox: in this case, it can be seen different typologies of gears connected together

KEYS

Keys are mechanical elements used to transmit the rotation from a shaft to another component. These kinds of mechanical elements need always a key seat and a keyway, to couple the two rotating elements.

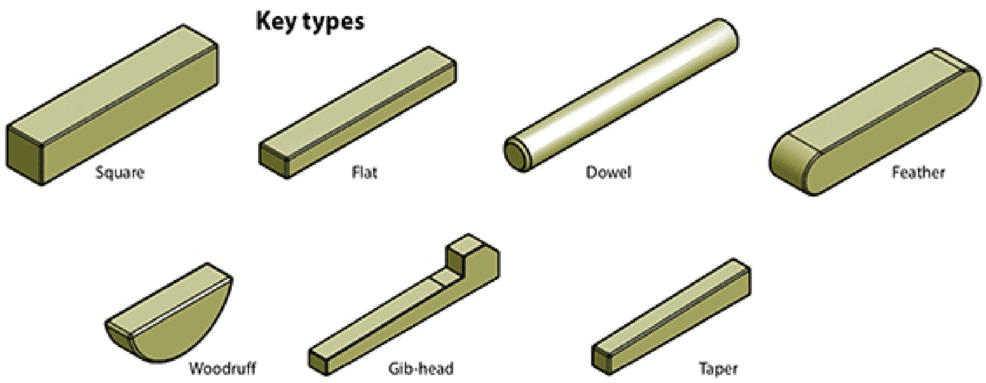
The keys permit to transmit even torques.



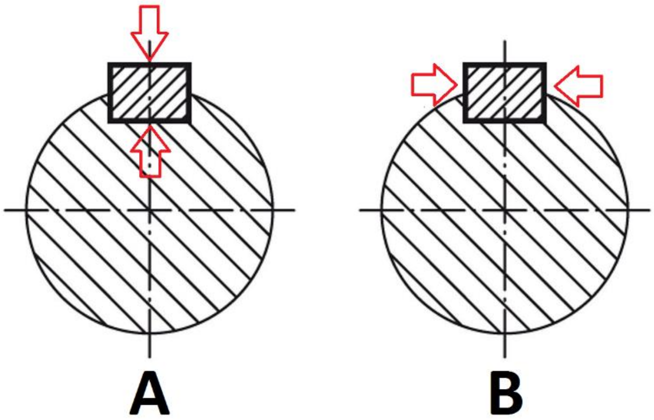


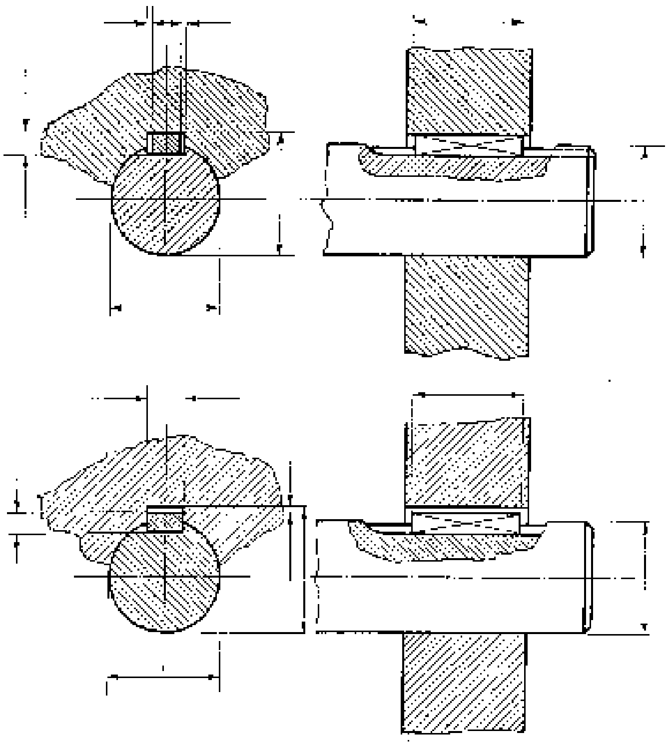
There are several types of keys, depending on the shape and by the kind of application.

Here following some examples.



An important kind of key is the parallel key. In fact, a key (in the figure, A) can transmit the movement through the radial friction between the two components: a parallel key (in the figure, B) instead transmits the torque thanks to the lateral contact.



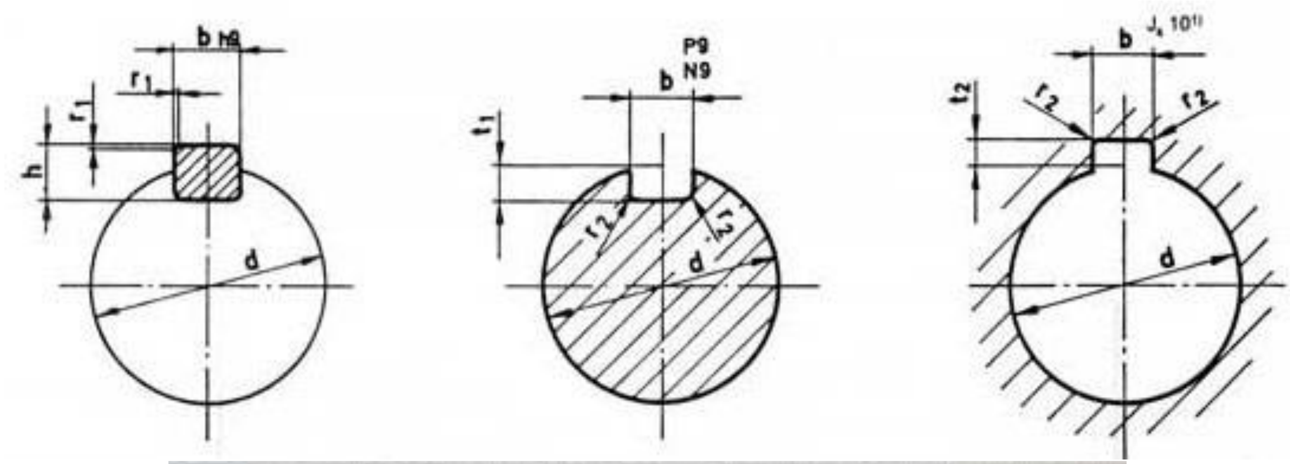


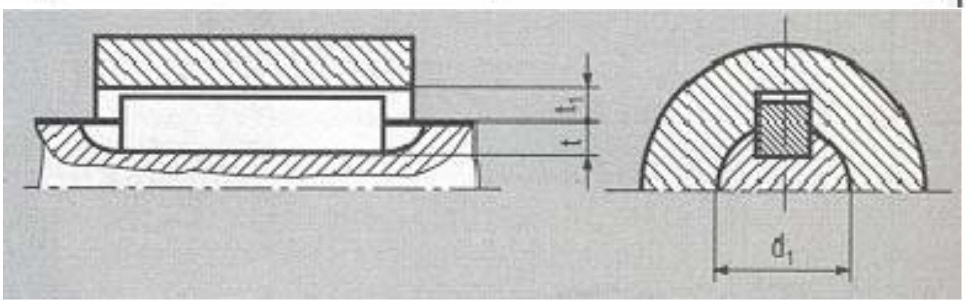
This lead to the following differences inside the drawing (in the figure on the left, above a *key* and below a *parallel key*).

As we can see on the left section, the key presents two gaps on the lateral surface: as explained before, this permit to transmit the torque using the friction between the two rotating elements.

Instead, a parallel key presents a gap between its top surface and the rotating element, as we can see on the two sections.

Here following some examples of how to dimension this kind of elements (tolerances are present):





Useful link: For more details, the following video is suggested:

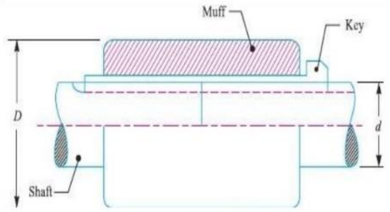
https://www.youtube.com/watch?v=S8Qmy4fGnnE

SHAFT COUPLINGS

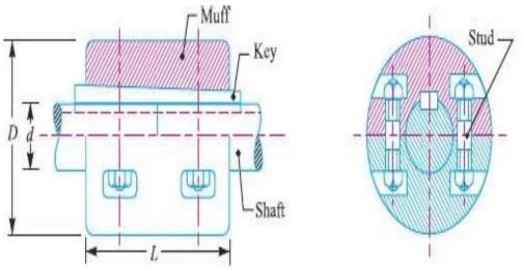
The shafts coupling is used to connect two successive rotating elements, allowing to connect different part of the driveline. There are several types of shaft coupling:

Rigid coupling: these couplings connect two shafts with no misalignment.

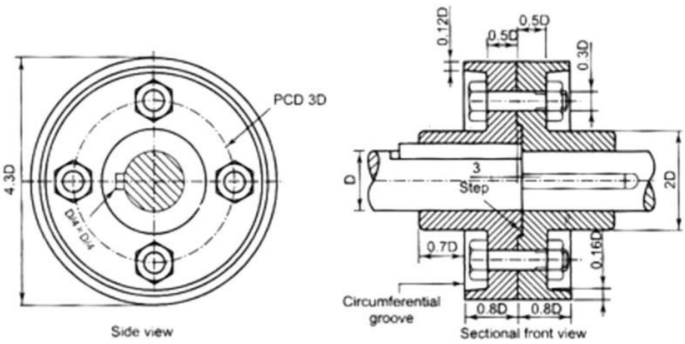
- *Sleeve or Muff*



- *Clamp or Compression*

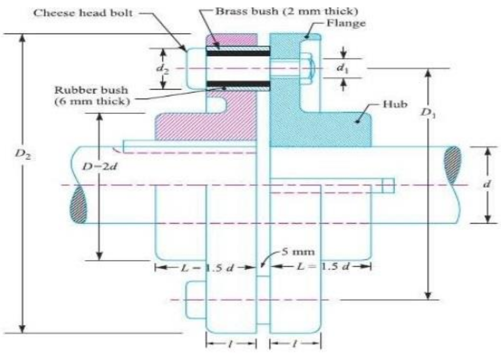


- *Flange*

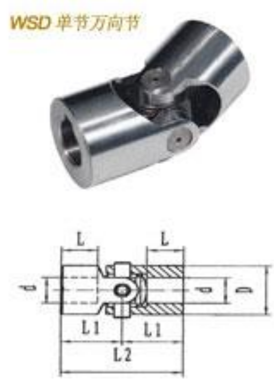


Flexible Coupling: these couplings connect shafts with some number of misalignments (axial, radial and angular) and can compensate them and some shock loads.

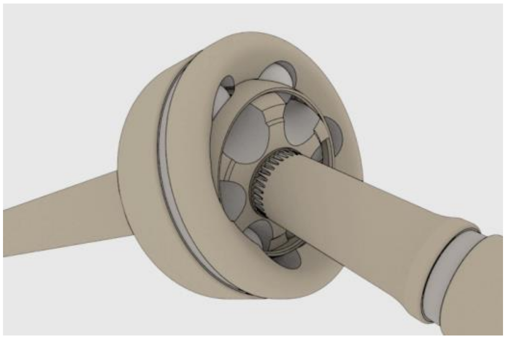
- *Bushed Pin*

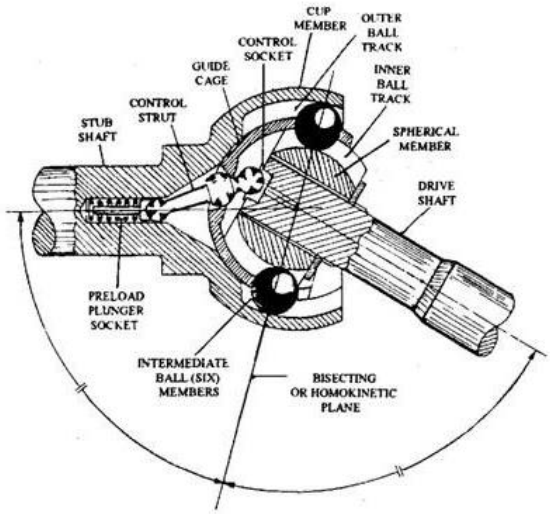


- *Universal:* these include the homokinetic joints, that allow two elements to rotate at the same angular velocity with some misalignment
    - *Cardan Joints*

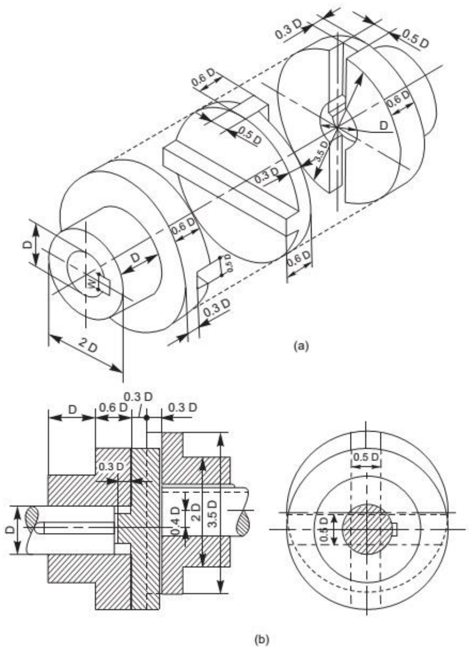


    - *Rzeppa Joints*

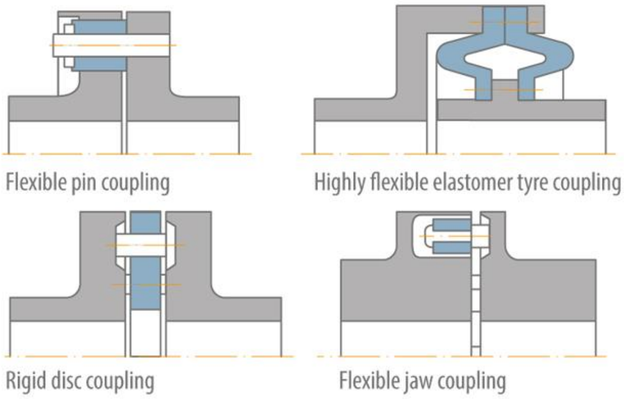




- *Oldham*



- *Other flexible couplings*



Safety Coupling: they restrict the transmitted torque between to elements and, over a certain amount, let them rotate freely.

Useful links:

A very good explanation of the shafts coupling is present on the following video:

https://www.youtube.com/watch?v=49l6ltnntFw

For more information and examples about the subject the following links are suggested:

https://www.ksb.com/centrifugal-pump-lexicon/shaft-coupling/191984/

https://www.slideshare.net/tparikh25/coupling-clutch-brake

(examples) http://www.daidoseimitu.co.jp/kata/rist/pdf/form_flex/coupling2.pdf

http://www.thomasnet.com/articles/hardware/coupling-types

USEFUL GENERAL LINK AND TEXTBOOKS

R.C. Juvinall , *Fundamentals of machine component Design,* Wiley

E.Chirone, S.Tornincasa, *Disegno Tecnico industriale* , il Capitello

https://ocw.mit.edu/index.htm

http://www.en.technisches-zeichnen.net/technical-drawing/basic-guides.php

https://engineering.tcnj.edu/

Some videos:

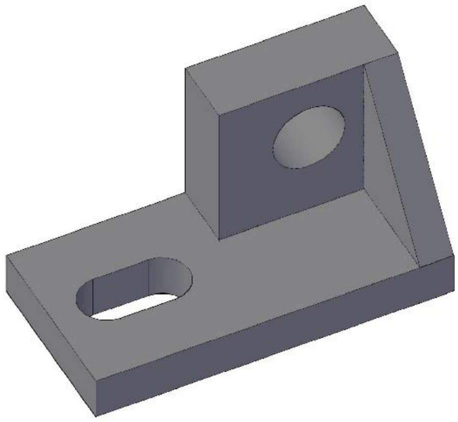
https://www.youtube.com/user/LearnEngineeringTeam

https://www.youtube.com/channel/UCqwJgEtC22XVVfBykI5baTg

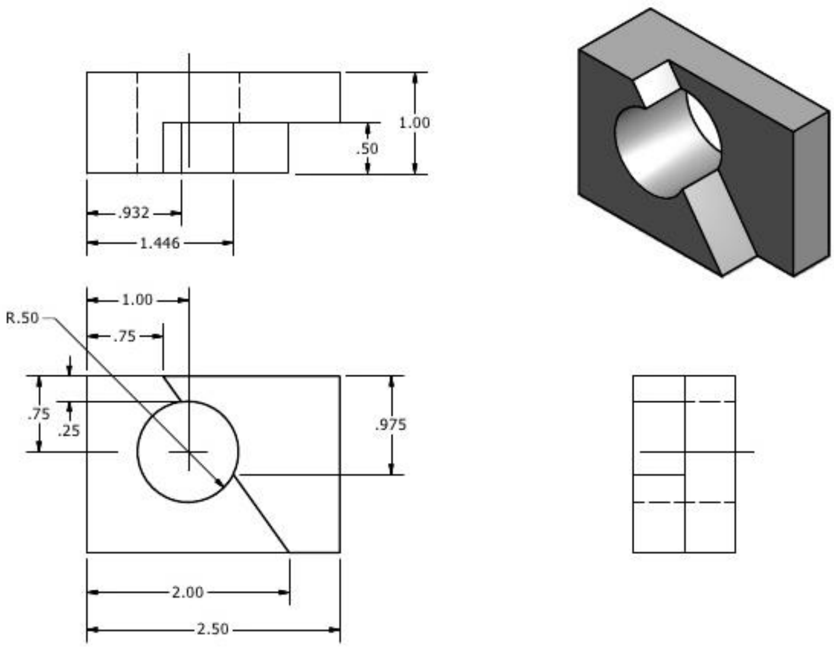
REVIEW EXERCISES

The right answers are indicated with a * on the end.

1. Are you able to make a first angle projection of the following figure? Make assumptions about the dimension and put them inside the drawing.



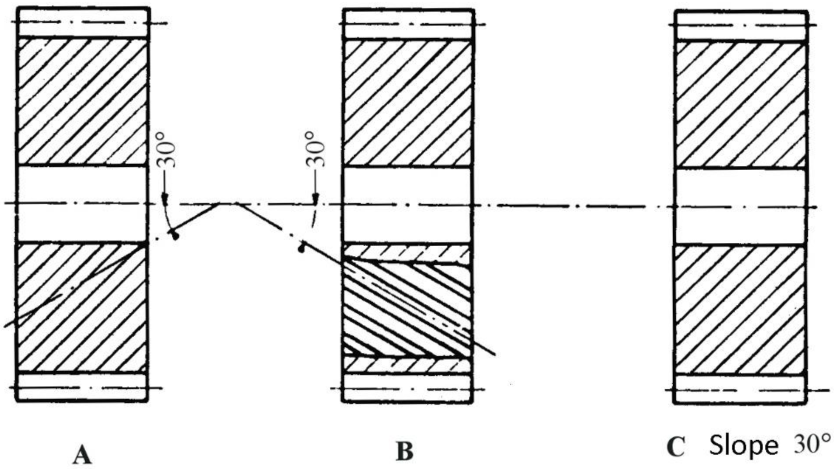
2. Are you able to find the errors inside this drawing? Say which of these assumptions are correct
    a. The hidden lines are wrong.
    b. The positions of the numbers respect to the dimension lines are wrong. *
    c. The axis lines are wrong. *
    d. The choice of the dimensions represented is wrong. *
    e. The hole dimension is represented correctly.



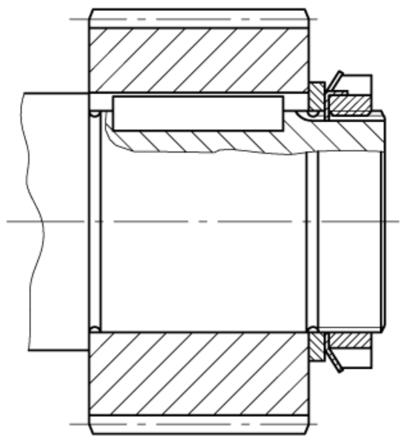
Try to redraw the same component and to correct the errors.

3. Which one of this answer is correct?

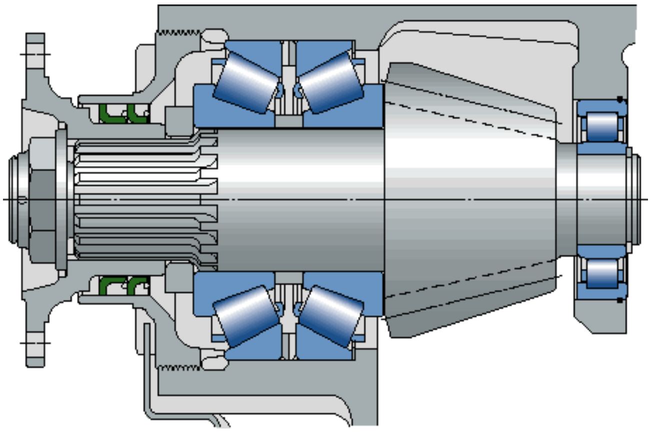
a. A is correct, B and C are wrong.
b. A and B are correct, C is wrong.
c. A and C are correct, B is wrong.
d. A is wrong, B and C are correct. *



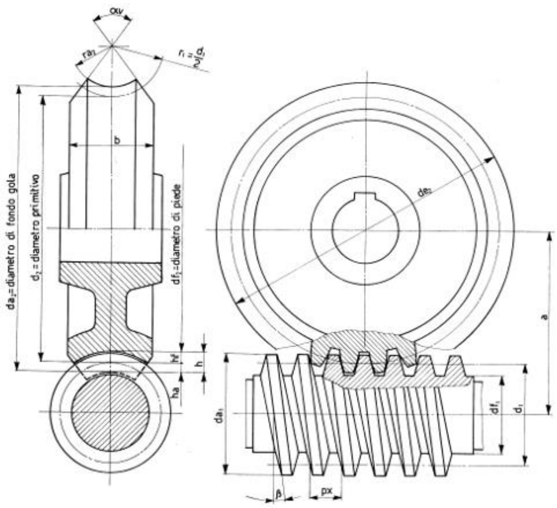
4. Are you able to recognize the following element?
   a. Bearing
   b. Key
   c. Parallel Key *
   d. Gib-head key



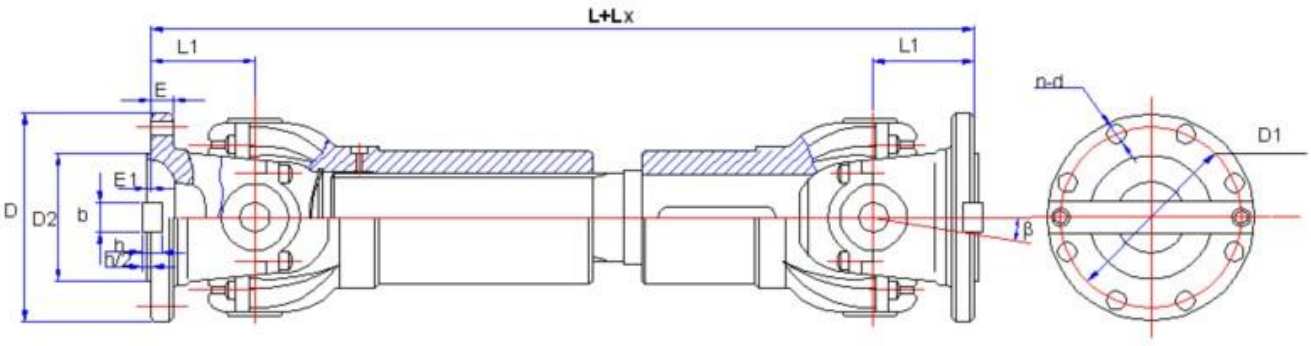
5. What do you know about bearings? Can you recognize this configuration?
   a. On the left axial bearings, on the right X configuration
   b. On the left X configuration, on the right O configuration
   c. On the left X configuration, on the right axial bearing
   d. On the left O configuration, on the right axial bearing *



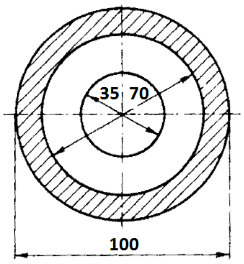
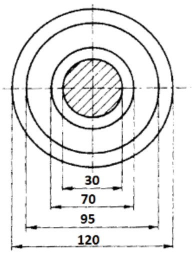
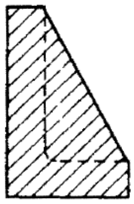
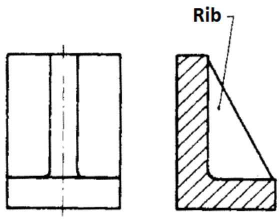
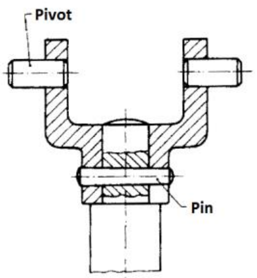
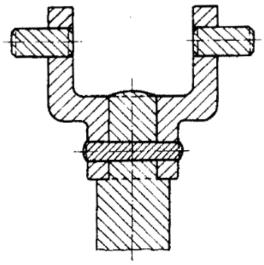
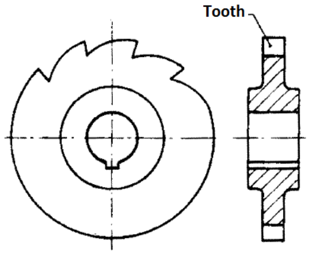
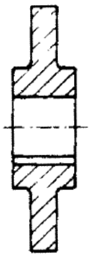
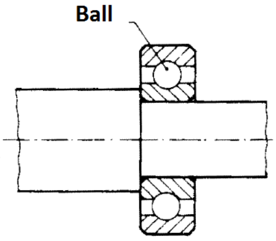
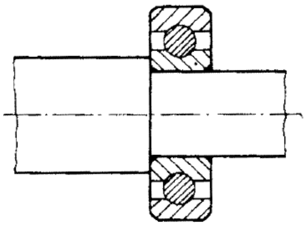
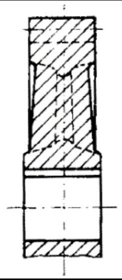
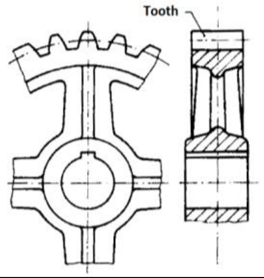
6. Which kind of gear is the following one?
   a. Rack and Pinion
   b. Worm and gear *
   c. Bevel
   d. Internal



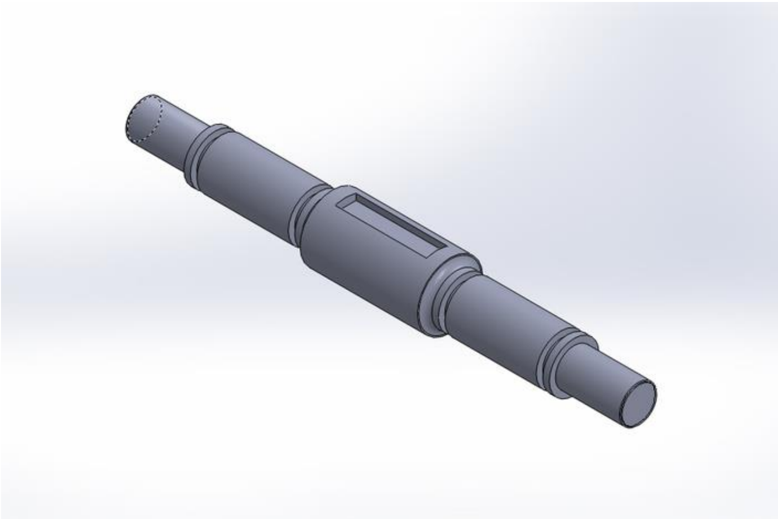
7. What do you know about shaft coupling? Are you able to recognize the following element?
   a. Flange Coupling
   b. Rzeppa joint
   c. Cardan Joint *
   d. Oldham Coupling



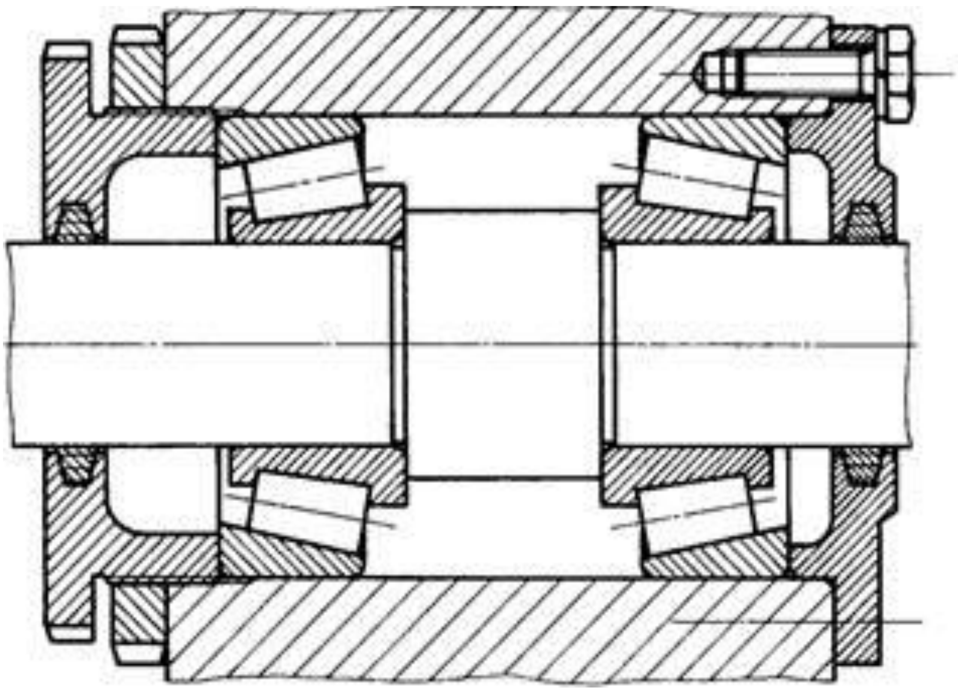
8. Find the errors in the following picture.

	A	B
1		
2		
3		
4		
5		
6		

9. Are you able to dimension the following shaft? (make assumptions about the dimensions)



10. Are you able to recognize the different elements present in the following drawing? Which of this one is not present in the drawing?
    a. Bearing
    b. Gear
    c. Shaft
    d. Spring *
    e. Screw
    f. Nut



MASS – SPRING – DAMPER SYSTEM

In classical mechanics, a harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force, F, proportional to the displacement, x.

If F is the only force acting on the system, the system is called a simple harmonic oscillator, and it undergoes simple harmonic motion: sinusoidal oscillations about the equilibrium point, with a constant amplitude and a constant frequency (which does not depend on the amplitude).

If a frictional force (damping) proportional to the velocity is also present, the harmonic oscillator is described as a damped oscillator. Depending on the friction coefficient, the system can: Oscillate with a frequency lower than in the non-damped case, and an amplitude decreasing with time (underdamped oscillator); Decay to the equilibrium position, without oscillations (overdamped oscillator).The boundary solution between an underdamped oscillator and an overdamped oscillator occurs at a particular value of the friction coefficient and is called "critically damped."

If an external time dependent force is present, the harmonic oscillator is described as a driven oscillator.

MSD-like systems are found throughout engineering in circuit design, robotics (PD control), vibrating system and automotive (cruise control, car suspensions) etc. If you are not familiar with this system, what you should do would be going to make up knowledge about mechanical system dynamics. Following links may help you to learn knowledge of mechanical system dynamics.

SIMPLE HARMONIC MOTION

Translating Systems	
Quantity	**Unit**
Mass - m	kg
Force - f	N
Length - x	m
Velocity - $\dot{x} = v$	m/s
Acceleration - $\ddot{x} = a$	m/s²
Spring Constant - k	N/m
Friction Coefficient - g	N-s/m

Chart 1 – translating systems physical quantities

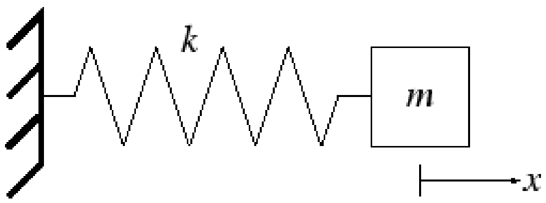


Figure 1 – An ideal mass-spring system

1. The Ideal Mass:
   a. The motion of an ideal mass is unaffected by friction or any other damping force
   b. The ideal mass is completely rigid
   c. By Newton's Second Law: $F = ma = m\,\frac{dv}{dt} = m\,\frac{d^2x}{dt^2}$

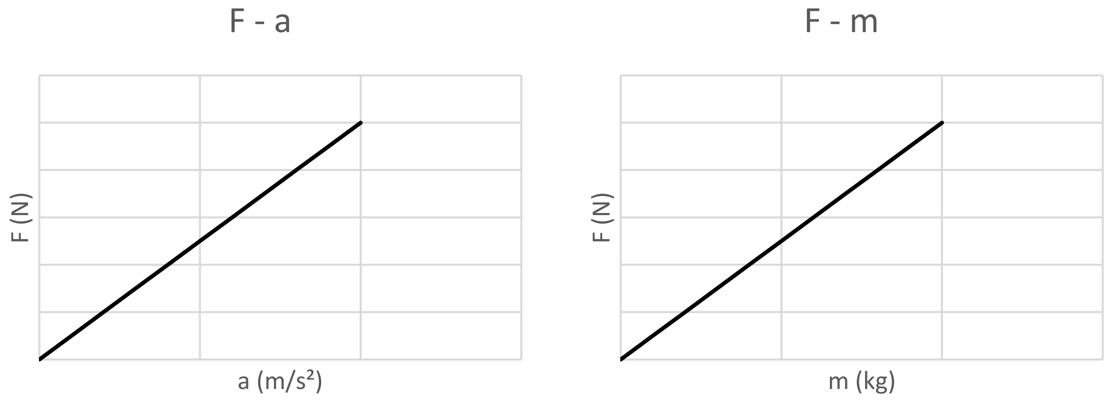


Figure 2 – relationship between F and a, F and m

2. The Ideal Spring:
   a. The ideal spring has no mass or internal damping
   b. Hooke's Law: $F = -kx$ (valid for small, non-distorting displacements)
   c. The spring's equilibrium position is given by $x = 0$
   d. A positive value of x produces a negative restoring force

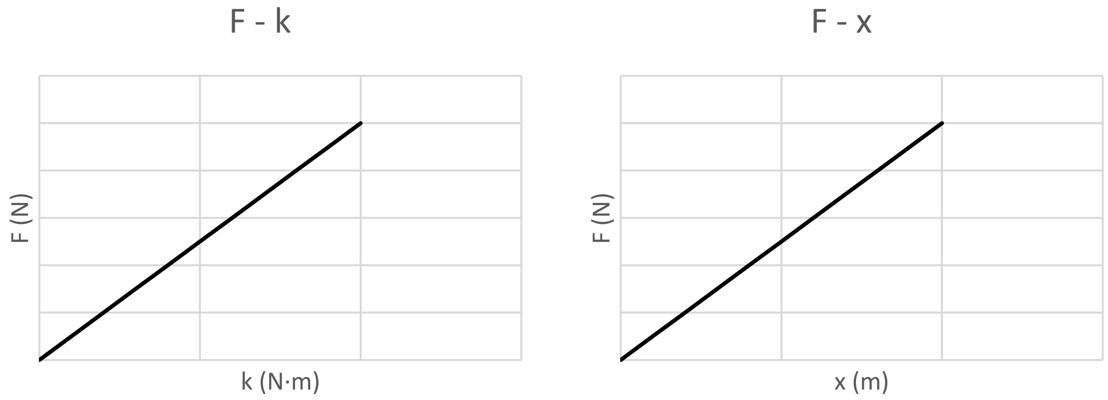


Figure 3 – relationship between F and k, F and x

3. The Ideal Mass-Spring System:
   a. System equation: $m\,\frac{d^2x}{dt^2} + kx = 0$
   b. There are no losses in the system, so it will oscillate forever
4. Energy in the Ideal Mass-Spring System

a. The potential energy E_p of the ideal mass-spring system is equal to the work done stretching or compressing the spring: $E_p = \int_0^x F\,dx = \int_0^x kx\,dx = \frac{1}{2}kx^2$

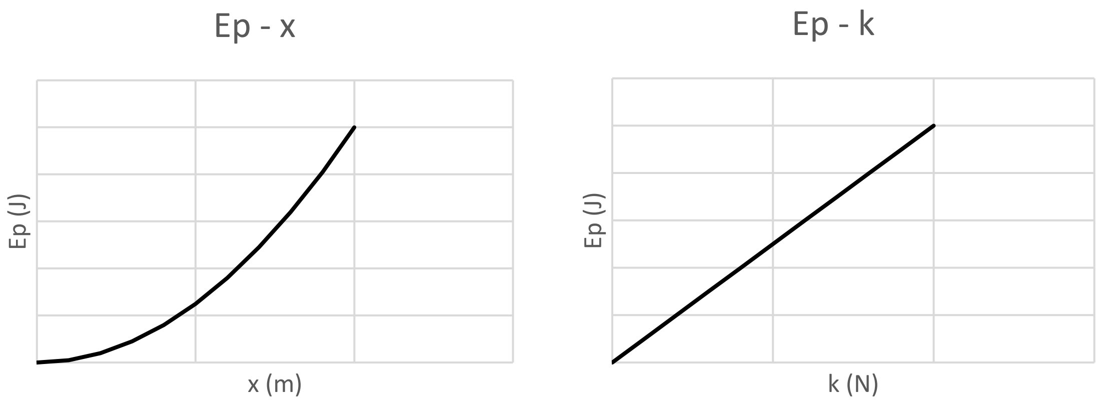


Figure 4 – Relationship between E_p and x, E_p and k

b. The kinetic energy E_k of the ideal mass-spring system is given by the motion of mass: $E_k = \frac{1}{2}mv^2$

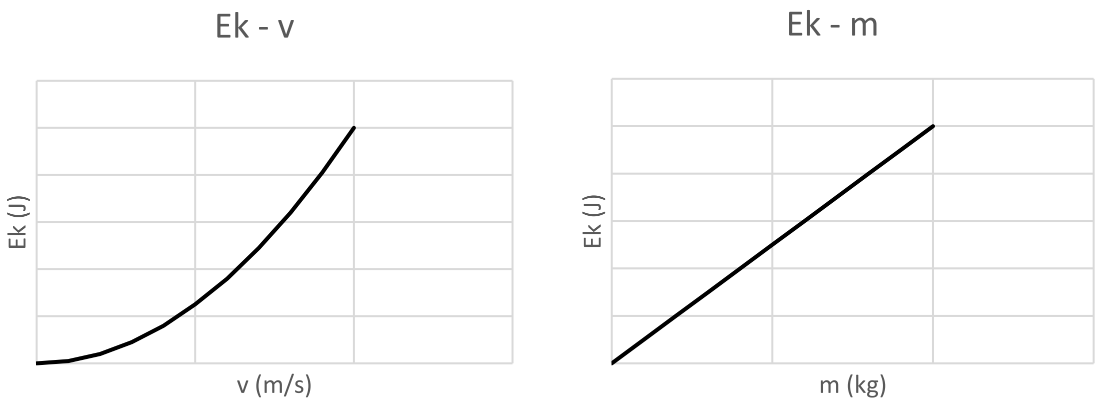


Figure 5 – Relationship between E_k and v, E_k and m

c. At the extremes of its displacement, the mass is at rest and has no kinetic energy. At the same time, the spring is maximally compressed or stretched, and thus stores all the mechanical energy of the system as potential energy.
d. When the mass is in motion and reaches the equilibrium position of the spring, the mechanical energy of the system has been completely converted to kinetic energy.
e. All vibrating systems consist of this interplay between an energy storing component and an energy carrying (“massy") component.

DAMPING

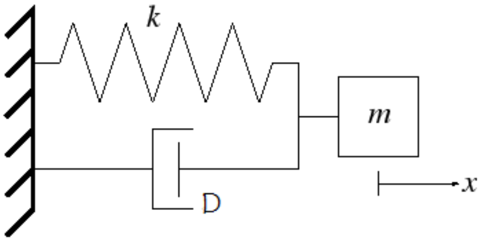


Figure 6 – An ideal mass-spring-damper system

1. The Ideal Mechanical Resistance:
   Force due to mechanical resistance or viscosity is typically approximated as being proportional to velocity: $F = Dv = D\frac{dX}{dt}$

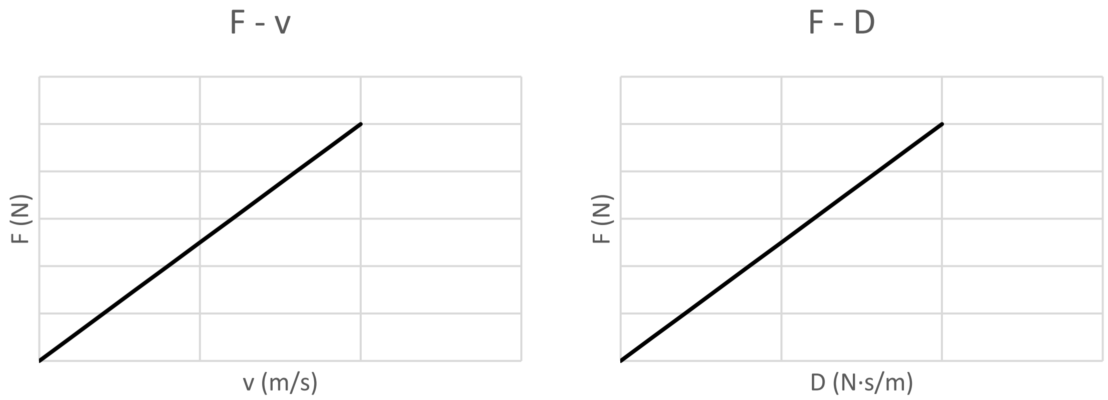
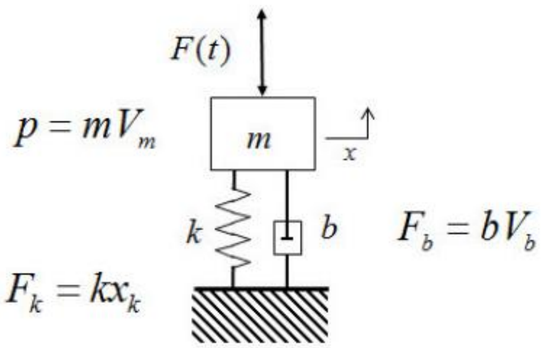


Figure 7 – relationship between F and v, F and D

2. The Ideal Mss-Spring-Damper System:
   System equation: $m\frac{d^2x}{dt^2} + D\frac{dx}{dt} + kx = 0$

EXAMPLE



$$m\ddot{x} = \sum F = -F_b - F_k + F(t)$$

$$m\ddot{x} = -b\dot{x} - kx + F$$

$$m\ddot{x} + b\dot{x} + kx = F$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m}$$

We can rewrite the equation to be $\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = u$, where u = F/m, $\omega_n$is the undamped natural frequency, and ζ is the damping ratio. From the undamped natural frequency define the undamped natural period $T_n = \frac{2\pi}{\omega_n}$.

The solutions are known for these cases, so it is worthwhile formulating model equations in the standard form, $\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = u(t)$.

Solutions for these cases are classified by ζ and a system is:

- underdamped if $\zeta < 1$
- overdamped if $\zeta > 1$
- critically damped if $\zeta = 1$

The effect of varying damping ratio on a second-order system can be seen in the following figure. It can be easy to notice the result of different underdamped, overdamped and critically damped.

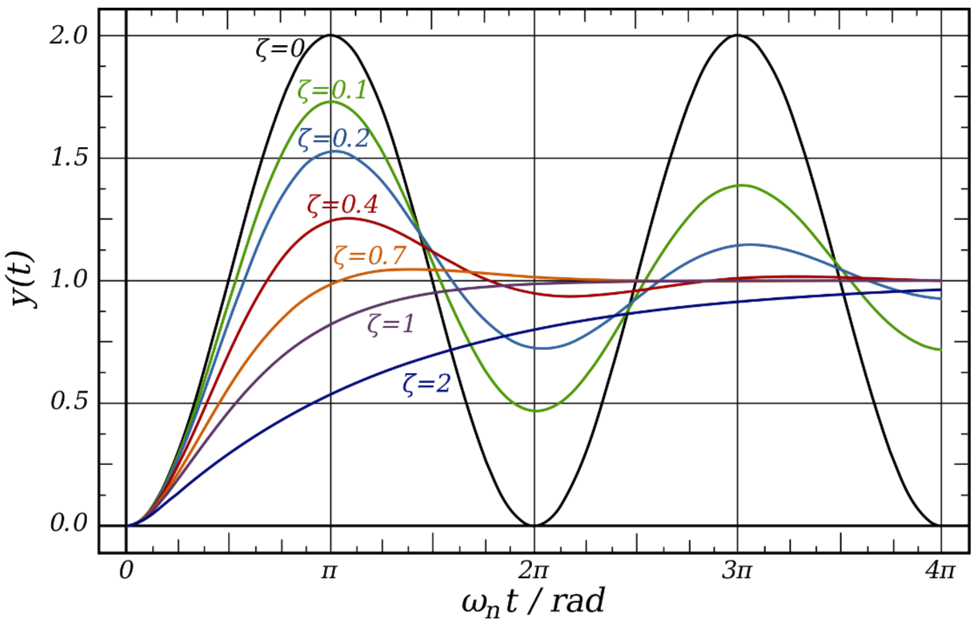


Figure 8 - The effect of varying damping ratio on a second-order system

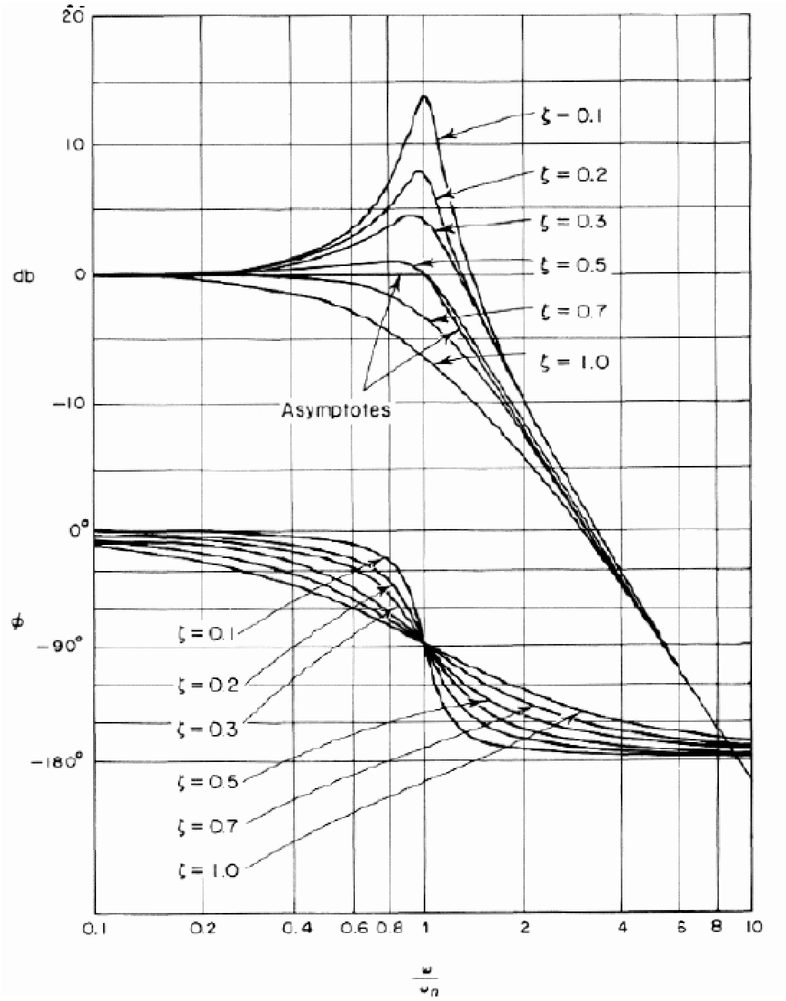


Figure 9 – Bode plot of different damping ratio

The bode plot for a linear, time-invariant system with transfer function H(s) (s being the complex frequency in the Laplace domain) consists of a magnitude plot and a phase plot.

The Bode magnitude plot is the graph of the function$|H(s = j\omega)|$ of frequency ω (with J being the imaginary unit.) The ω-axis of the magnitude plot is logarithmic and the magnitude is given in decibels, i.e., a value for the magnitude |H| is plotted on the axis at $20 \log_{10} |H|$.

The Bode phase plot is the graph of the phase of the transfer function arg $(H(s = j\omega))$ as a function of ω commonly expressed in degrees. The phase is plotted on the same logarithmic ω-axis as the magnitude plot, but the value for the phase is plotted on a linear vertical axis.

More detail about converting a differential equation from time-domain to Laplace-domain, please check related electric review part.

TORSIONAL MECHANISM

As with translating mechanical systems, there are three fundamental physical elements that comprise rotating mechanical system: inertia elements, springs and friction elements.

EXAMPLE

This is an example of a mass-spring-damper system with rotating mechanical elements, which can be expressed a simple model of transmission in the car.

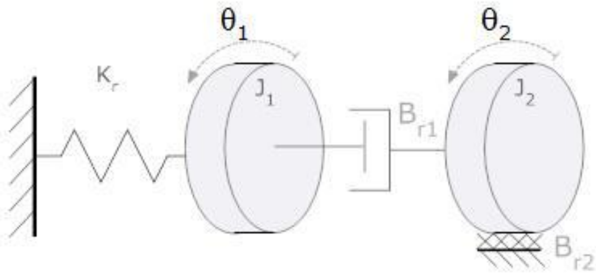


Figure 10 - rotating system

Three elements were introduced: springs, friction elements and inertial elements (masses). An ideal linear spring has no mass and a linear relationship between force and elongation. For viscous friction, there is a linear relationship between force and velocity. Friction may either be between two surfaces (depicted as hash marks) or between two objects (depicted as a dashpot). An ideal dashpot is also massless. Masses have a linear relationship between force and acceleration.

Rotating Systems	
Quantity	**Unit**
Moment of Inertia - J	$Kg\cdot m^2$
Torque - τ	$N\cdot m$
Angle - θ	rad
Angular velocity - $\dot{\theta} = \omega$	rad/s
Angular acceleration - $\ddot{\theta} = \alpha$	rad/s^2
Spring Constant - Kr	$N\cdot m/rad$
Friction Coefficient - Br	$N\cdot m\cdot s/rad$

Chart 2 – Rotating systems physical quantities

Constitutive Equations for Rotating Mechanical Elements:

1. Spring: A rotational spring is an element that is deformed (wound or unwound) in direct proportion to the amount of torque applied. Ideal springs have no inertia. The relationship between torque, spring constant and angle is given by F = Kr θ

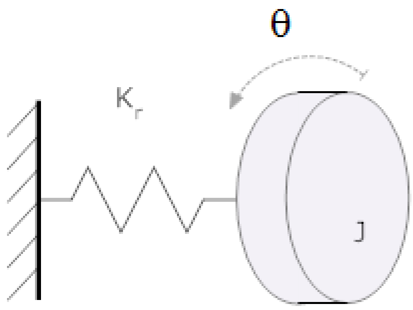


Figure 10 - rotating spring

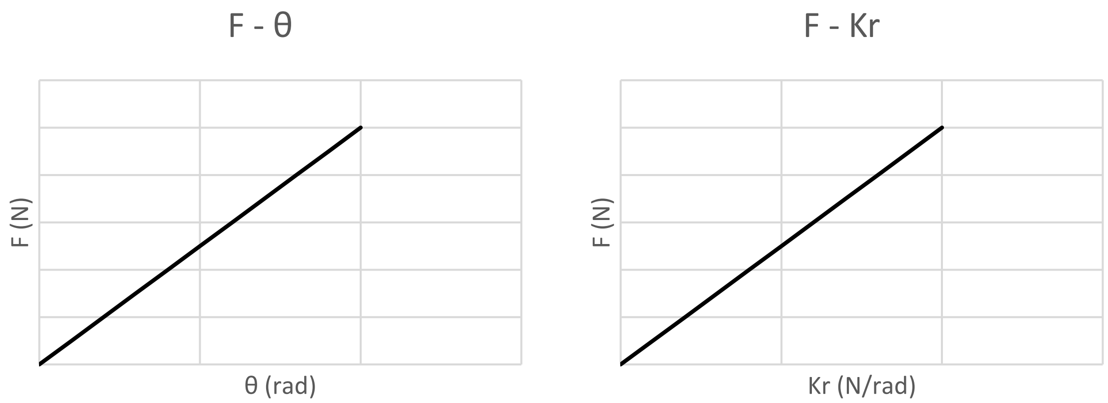


Figure 11 – relationship between F and θ, F and Kr

2. Friction: As with the translating systems, friction is the most difficult of the three elements to model accurately and we will generally only consider viscous friction. The constitutive equation relating angular velocity, torque and friction coefficient is: Fc = Br $\dot{\theta}$ = Brω

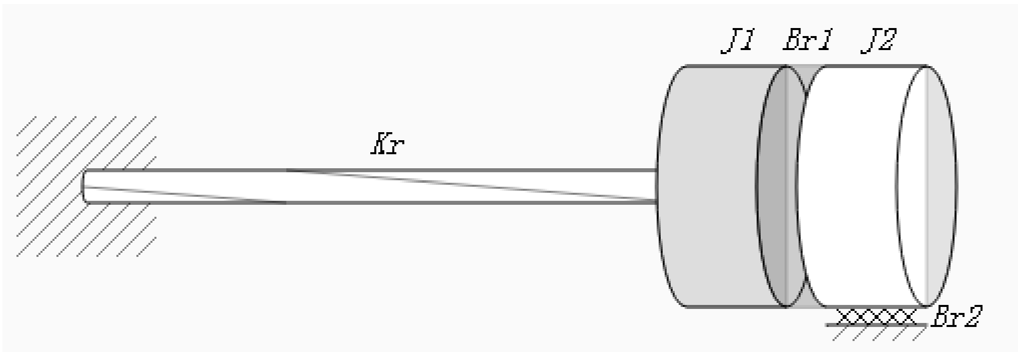


Figure 12 - rotating friction

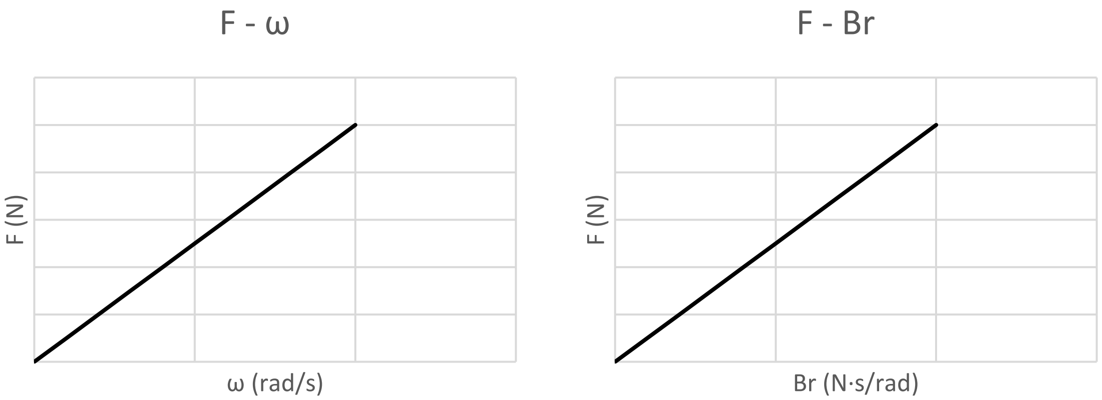


Figure 11 – relationship between F and ω, F and Br

3. Inertial: In rotating mechanical systems, the inertia elements are masses that rotate and are characterized by a moment of inertia. The moment of inertia for some common shapes is given below. The relationship between torque, moment of inertia and angular acceleration is given by $\tau = J\ddot{\theta} = J\alpha$

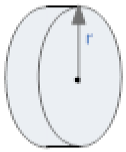
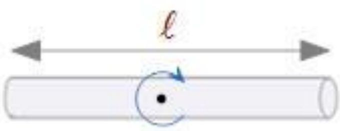
Shape	Image	Moment of Inertia, J
Cylinder, radius=r, mass=m Rotating about center axis		$\frac{1}{2}mr^2$
Solid Sphere, radius=r, mass=m Rotating about center		$\frac{2}{5}mr^2$
Uniform Rod, length=ℓ, mass=m Rotating about end		$\frac{1}{3}m\ell^2$
Uniform Rod, length=ℓ, mass=m Rotating about center		$\frac{1}{12}m\ell^2$
Mass at end of massless rod, length=ℓ, mass=m Rotating about end		$m\ell^2$

Chart 3 – different shape of inertial

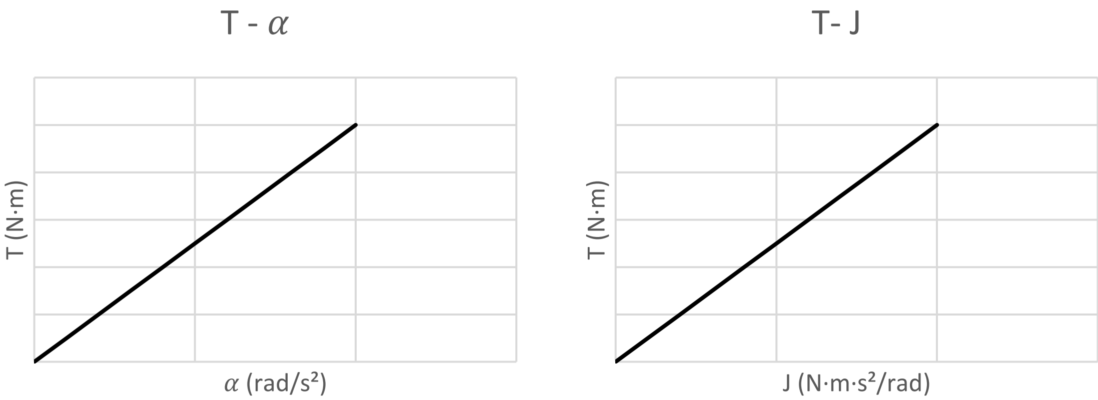


Figure 11 – Relationship between F and ω, F and Kr

4. Potential Energy: stored in a rotational spring with spring constant: $E_p = \frac{1}{2} K_r \theta^2$

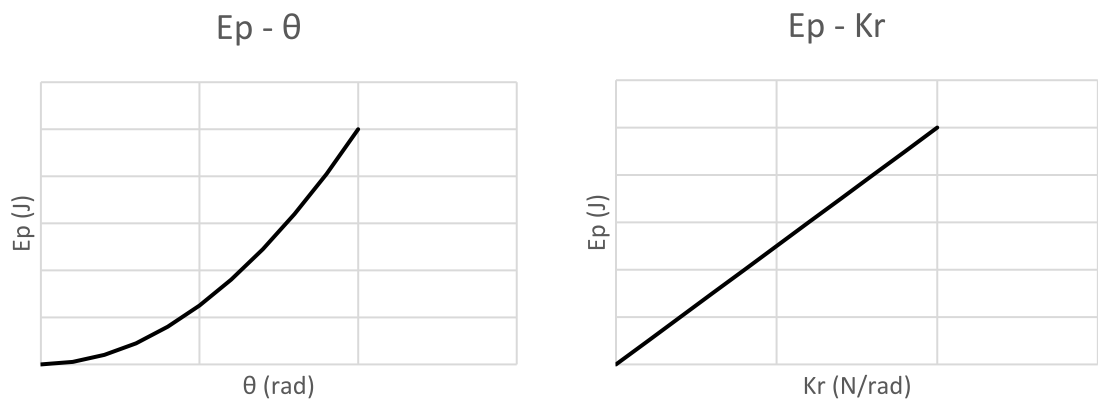


Figure 12 – Relationship between E_p and θ, E_p and Kr

5. The kinetic energy: stored in a rotating mass with a moment of inertia and angular velocity: $E_k = \frac{1}{2} J \dot{\theta}^2 = \frac{1}{2} J \omega^2$

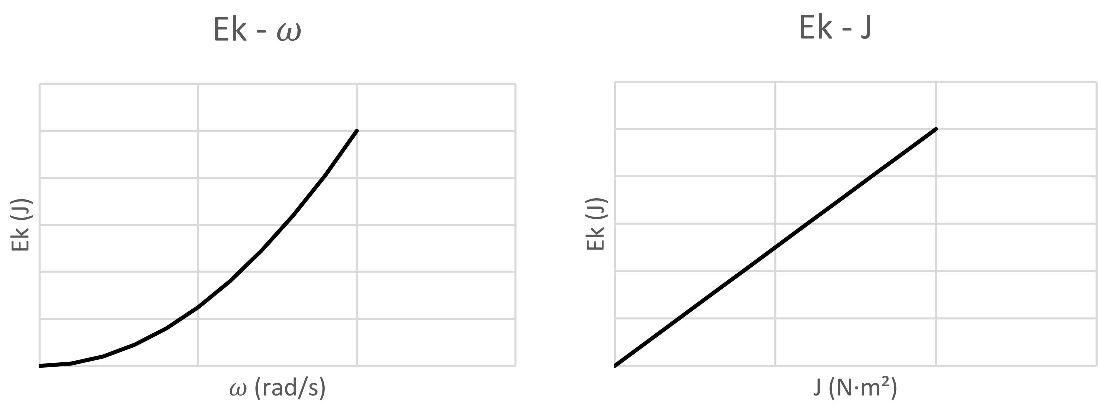


Figure 13– Relationship between E_k and ω, E_k and J

6. Dissipated energy: in a friction element with angular velocity is given by: $E_d = B_r \int_0^t \omega dt = \frac{1}{2} B_r \omega^2$

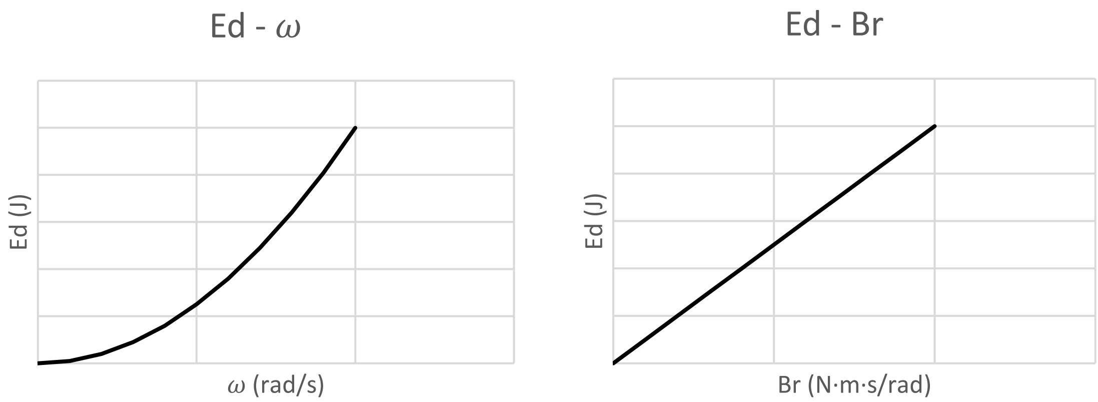


Figure 14– Relationship between E_d and ω, E_d and Br

7. Power: in rotating systems is given by $P = \tau \cdot \omega$, where τ is the torque and ω is the angular velocity.

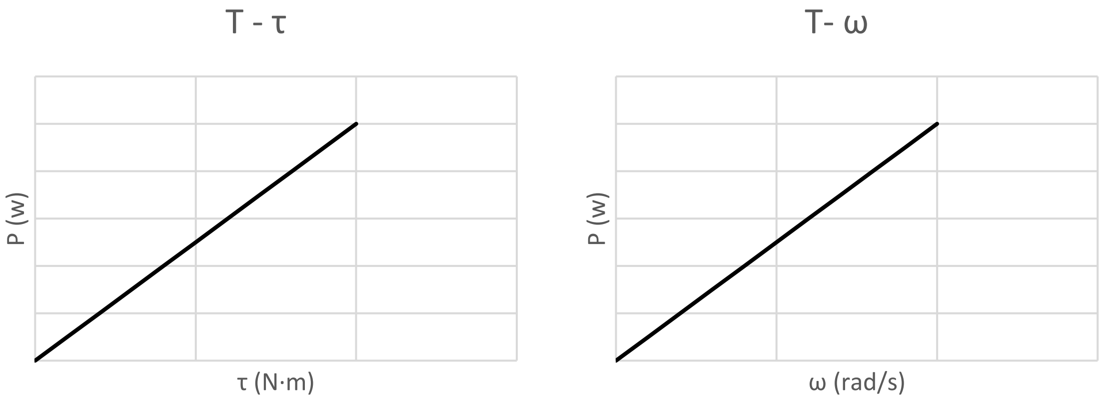


Figure 15 – relationship between P and τ, P and ω

Useful links:

https://www.youtube.com/watch?v=wBf3IkiJuog

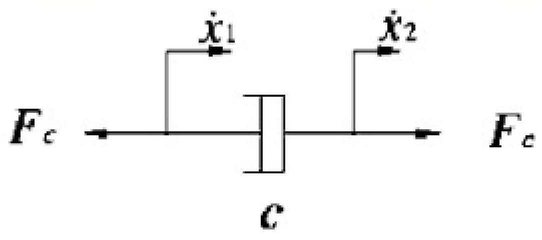
https://www.youtube.com/watch?v=b6JqPKobAXY

https://www.youtube.com/watch?v=3i8rCIeXzns

http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=SystemModeling

III. EXERCISE

1. What is the physical meaning of damping? Find the force and dissipative energy equation about the following system.



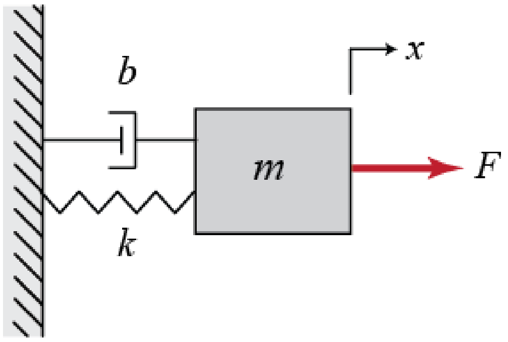
Answer:

Damping is a component representing a relationship between force and velocity, is a measure of the system damping characteristics, reflecting the dissipative energy during vibration process of the system.

$$F_c = c \cdot (\dot{x}_1 - \dot{x}_2)$$

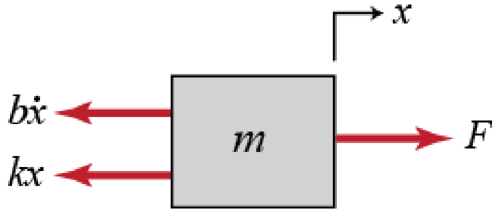
$$D = \frac{1}{2} \cdot c \cdot (\dot{x}_1 - \dot{x}_2)^2$$

2. Consider a simple system with a mass that is separated from a wall by a spring and a dashpot. Only horizontal motion and forces are considered.



Answer:

The completed free body diagram is shown below:



Now we proceed by summing the forces and applying Newton's second law,

$$\Sigma F = F - b\dot{x} - kx = m\ddot{x}$$

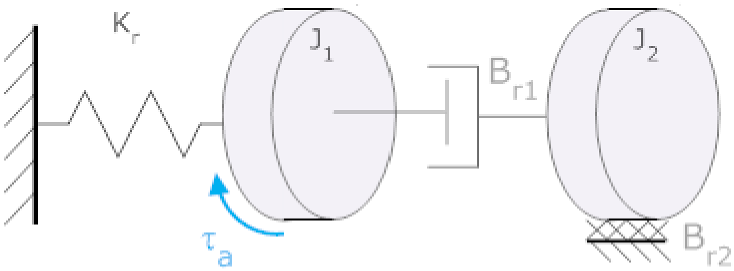
The Laplace transform for this system assuming zero initial conditions is: (more detail about Laplace transform please to look up another review)

$$F(s) = ms^2X(s) + bsX(s) + kX(s)$$

and therefore, the transfer function from force input to displacement output is

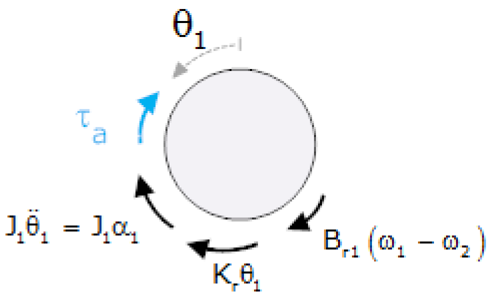
$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

3. In the system shown one flywheel (J1) is attached trough a flexible shaft (Kr) to ground (the unmoving wall) and has an applied torque τ a. A second flywheel (J2) is driven by friction between the two flywheels (Br1). The second flywheel also has friction to the ground (Br2). Derive equations of motion for the system shown.



Answer:

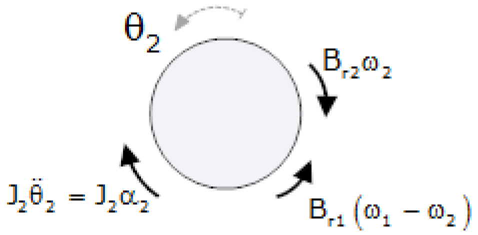
For the free body diagram at θ_1 there are 4 torques acting: the external torque, clockwise; the torque due to K_r; the torque due to B_{r1}; the torque due to J_1.



$$\tau_a + J_1\alpha_1 + B_{r1}(\omega_1 - \omega_2) + K_r\theta_1 = 0$$

$$J_1\ddot{\theta}_1 + B_{r1}\dot{\theta}_1 + K_1\theta_1 - B_{r1}\dot{\theta}_2 = -\tau_a$$

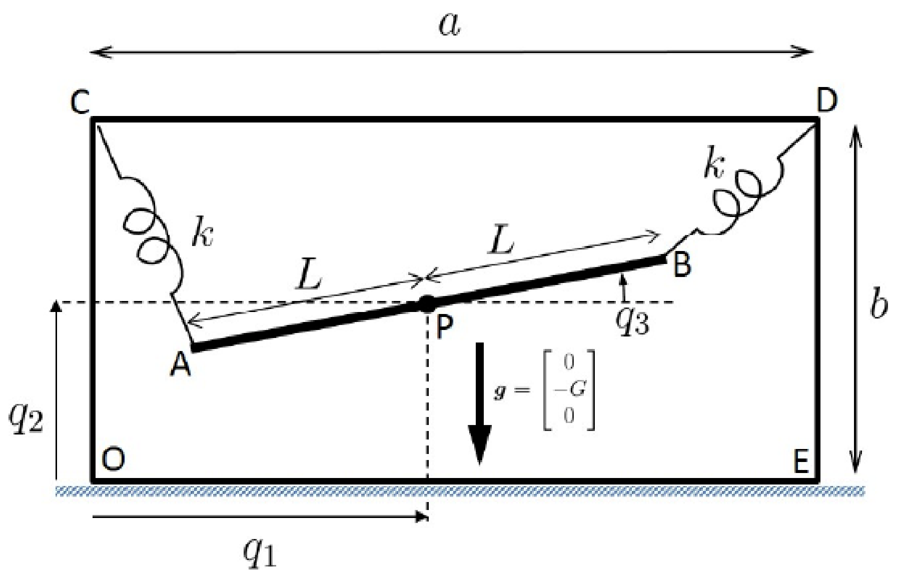
For the free body diagram at θ_2 there are 3 torques acting: the torque due to B_{r2}; the torque due to B_{r1}; the torque due to J_2 clockwise.



$$J_2\alpha_2 + B_{r2}\omega_2 - B_{t1}(\omega_1 - \omega_2) = 0$$

$$J_2\ddot{\theta}_2 + (B_{r2} + B_{r1})\dot{\theta}_2 - B_{r1}\dot{\theta}_1 = 0$$

4. The planar system represented in figure consists of a fixed box OCDE with a suspended thin rigid bar APB (P is the middle point); the bar mass is M, the bar length is 2L, and the bar principal inertia matrix with respect to P around the z-axis is $\Gamma_z = \frac{1}{3}mL^2$. The points A and B are connected to C and D by elastic springs with equal elastic constant k and the rest length $l_s = 0$. The generalized coordinates$q_i, i = 1,2,3$ are indicated in Figure. The inertial reference frame origin is located in O. Compute the total energy equation in this system.



Answer:

Since no dissipative function and no external forces are present, it is only necessary to compute the kinetic co-energy and the potential energy.

- Kinetic co-energy: $\mathrm{K} = \frac{1}{2} m v^2 + \frac{1}{2}\, \omega \Gamma \omega$

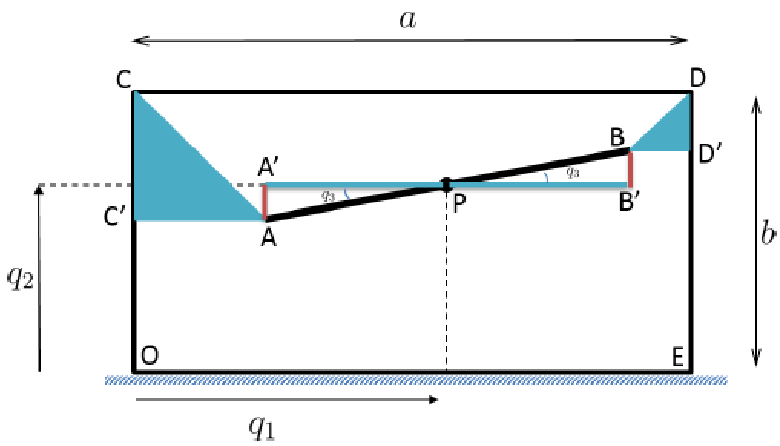
  Since the position of the point P is simply $\mathrm{P} = [\mathrm{q}_1\ \mathrm{q}_2\ 0]^T$, the velocity $\mathrm{v} = [\dot{q}_1\ \dot{q}_2\ 0]^T$ and the angular velocity is $\omega = [0\ 0\ \dot{q}_3]^T$, we can conclude that

$$\mathrm{K} = \frac{1}{2} m \dot{q}_1{}^2 + \frac{1}{2} \cdot \frac{1}{3} m L^2 \dot{q}_3{}^2$$

- Potential energy: $P = P_g + P_e$

$$\mathrm{P_g} = mG$$

The computation of $\mathrm{P_e}$ is a little more complicated Look at the figure of the system in a generic configuration



$$\mathrm{P_e} = \frac{1}{2} k (CA^2 + BD^2)$$

From the Pitagora's theorem

$$CA^2 = CC'^2 + C'A^2$$
$$BD^2 = BD'^2 + D'D^2$$

From the figure, we easily find the following relations

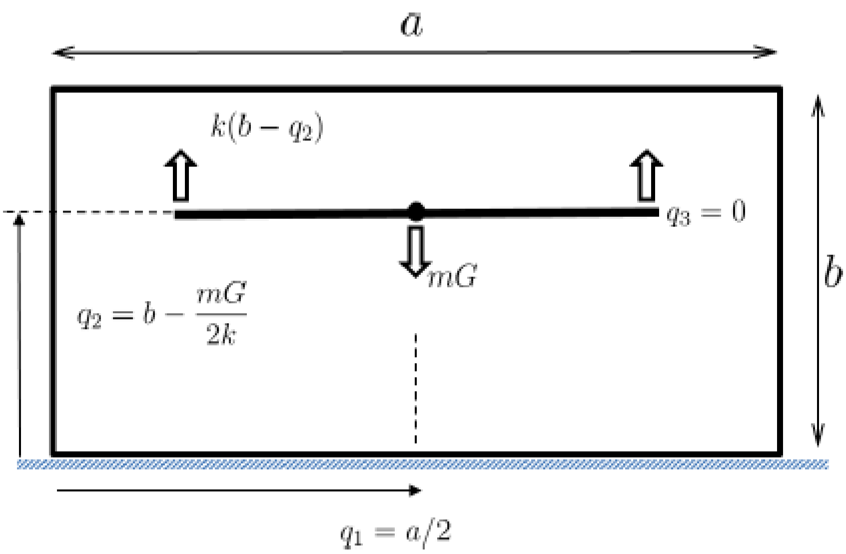
$$q_1 + L\cos(q_3) + BD' = a \qquad BD' = a - q_1 - L\cos(q_3)$$
$$C'A + L\cos(q_3) = q_1 \qquad C'A = q_1 - L\cos(q_3)$$
$$q_2 + L\sin(q_3) + DD' = b \qquad DD' = b - L\sin(q_3) - q_2$$

$q_2 - Lsin(q_3) + CC' = b \qquad CC' = b - q_2 - Lsin(q_3)$

Hence, using the abbreviated symbols, where c_3 is $\cos(q_3)$ and s_3 is $\sin(q_3)$

$$CA^2 = (q_1 - Lc_3)^2 + (q_2 - b - Ls_3)^2$$
$$BD^2 = (q_1 - a + Lc_3)^2 + (q_2 - b + Ls_3)^2$$
$$P_e = \frac{1}{2}k[(q_1 - Lc_3)^2 + (q_2 - b - Ls_3)^2 + (q_1 - a + Lc_3)^2 + (q_2 - b + Ls_3)^2]$$

The system in equilibrium configuration when $\ddot{q} = \dot{q} = 0$. The following static equilibrium result can be found out by means of Lagrange equations (this method will be introduced in course named modelling and simulation of mechatronic systems).



FLUID SYSTEM

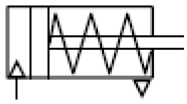
A pneumatic and hydraulic system is called fluid system. Fluid mechanics is a branch of physics concerned with the mechanics of fluids (liquids, gases, and plasmas) and the forces on them. Fluid mechanics has a wide range of applications, including for mechanical engineering, civil engineering, chemical engineering, geophysics, astrophysics, and biology. Fluid mechanics can be divided into fluid statics, the study of fluids at rest; and fluid dynamics, the study of the effect of forces on fluid motion. It is a branch of continuum mechanics, a subject which models matter without using the information that it is made out of atoms; that is, it models matter from a macroscopic viewpoint rather than from microscopic. Fluid mechanics, especially fluid dynamics, is an active field of research with many problems that are partly or wholly unsolved. Fluid mechanics can be mathematically complex, and can best be solved by numerical methods, typically using computers.

LINEAR ACTUATOR

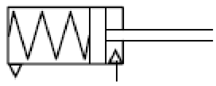
A linear actuator is an actuator that creates motion in a straight line, in contrast to the circular motion of a conventional electric motor. Linear actuators are used in machine tools and industrial machinery, in computer peripherals such as disk drives and printers, in valves and dampers, and in many other places where linear motion is required. Hydraulic or pneumatic cylinders inherently produce linear motion. Many other mechanisms are used to generate linear motion from a rotating motor.

1. Single acting:

Single acting sprung in-stroked

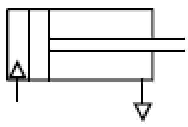


Single acting sprung out-stroked

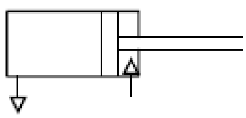


2. Single acting without spring:

Single acting normally in-stroked
external force returns

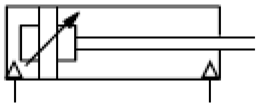


Single acting normally out-stroked
external force returns

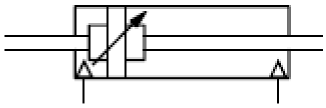


3. Double acting:

Double acting adjustable cushions

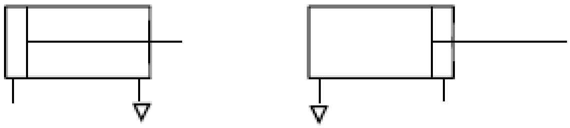


Double acting through rod



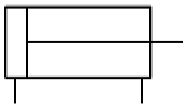
4. Simplified cylinder symbols:

Single acting load returns

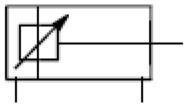


Single acting spring returns

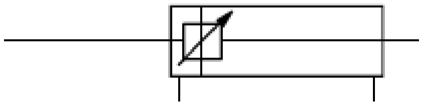
Double acting non-cushioned



Double acting adjustable cushions



Double acting through rod



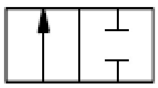
Useful links:

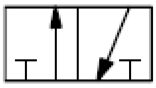
https://en.wikipedia.org/wiki/Pneumatic_cylinder

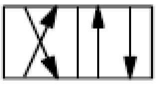
https://en.wikipedia.org/wiki/Hydraulic_cylinder

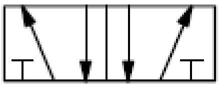
CONTROL VALVE

Function 2/2



Function 3/2 

Function 4/2 

Function 5/2 

1. Valve with 2 ports and 2 positions. Can be used when downstream fluid (from actuator) is not exhausted towards the valve.

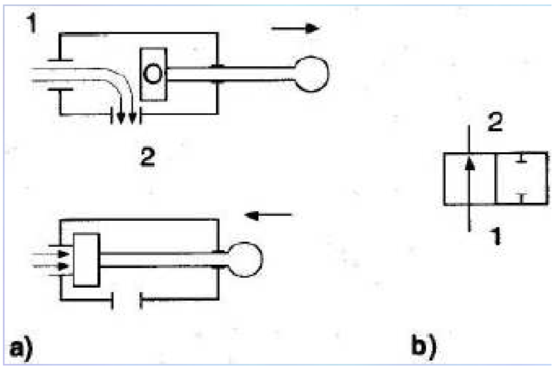


Figure 16 – 2/2 valve

2. Valve with 3 ports and 2 positions. Port 1 connected to supply, 2 connected to the outlet (towards the actuator), 3 connected to exhaust. Port 2 can be connected to either supply 1 or exhaust 3.
   -Normally open: those that are open from the supply (1) to the working port (2) when no operating signal is applied;
   -Normally closed: those that are closed from the supply (1) to the working port (2) when no operating signal is applied; (working port (2) is connected to the exhaust (3)).

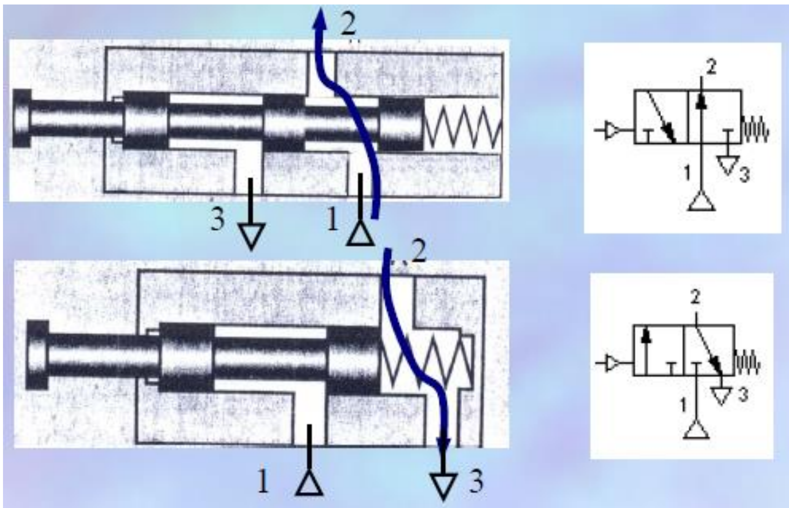


Figure 17 - normally open and normally closed 3/2 valve

3. Valve with 4 ports and 2 positions. In the scheme a 4/2-way monostable valve obtained by the combination of two 3/2-way valves, one valve normally closed and the other normally open. The solution is of closed center type: when the pneumatic control acts at the control port, 1 to 2 and 4 to 3 are closed. Then valve poppets are pressed further against the reset springs and the passages from 1 to 4 and from 2 to 3 are opened.

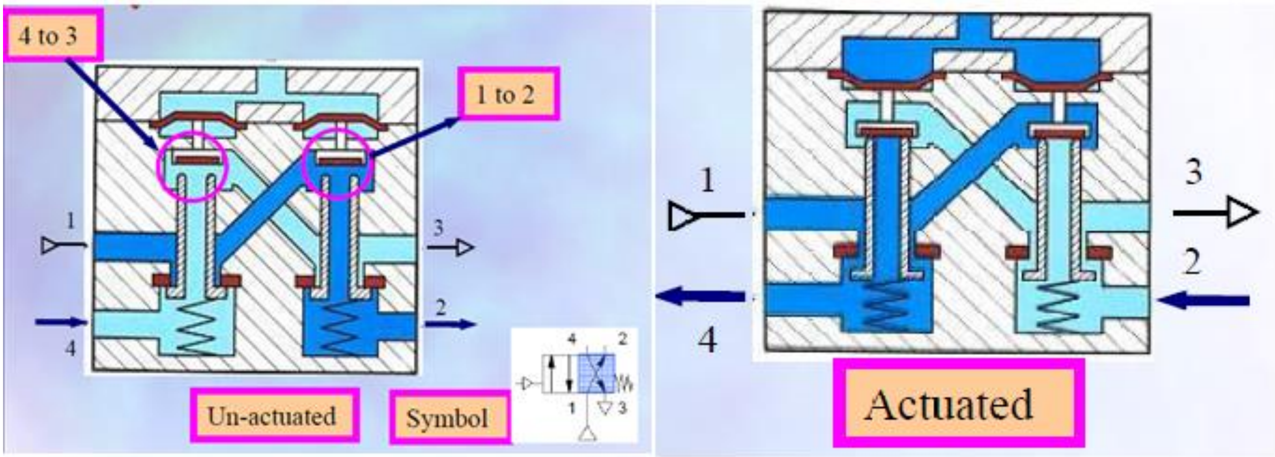


Figure 18 - 4/2 valve

4. Valve with 5 ports and 2 positions. T his type of valve has 5 ports: port 1 connected to supply, 2 and 4 connected to the outlet (towards the actuator), 3 and 5 connected to exhaust. Supply port 1 can be connected to either working port 2 or working port 4. When port 2 is connected to supply 1, port 4 is connected to exhaust 5 and vice versa.

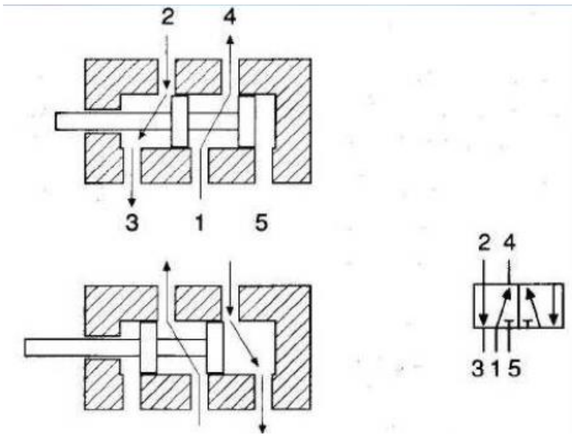


Figure 19 - 5/2 valve

EXAMPLE

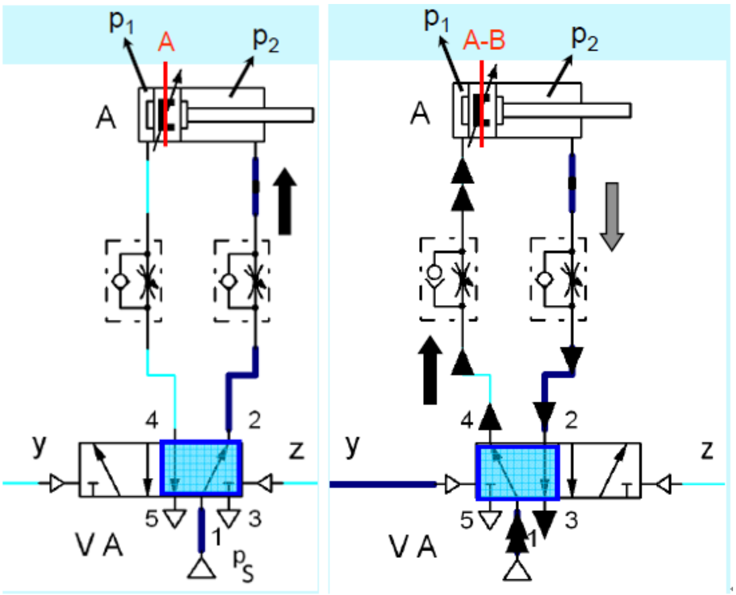


Figure 20 – initial condition and switched condition

Initial condition: cylinder A completed retracted and ready for extension.

The valve VA is in the position shown (in light blue the current position), waiting for the activation of the control signal y. The valve VA is switched by the control signal y but whereas the pressure differential $p_1 - p_2 = \Delta p$ the piston does not move (velocity is still v=0).

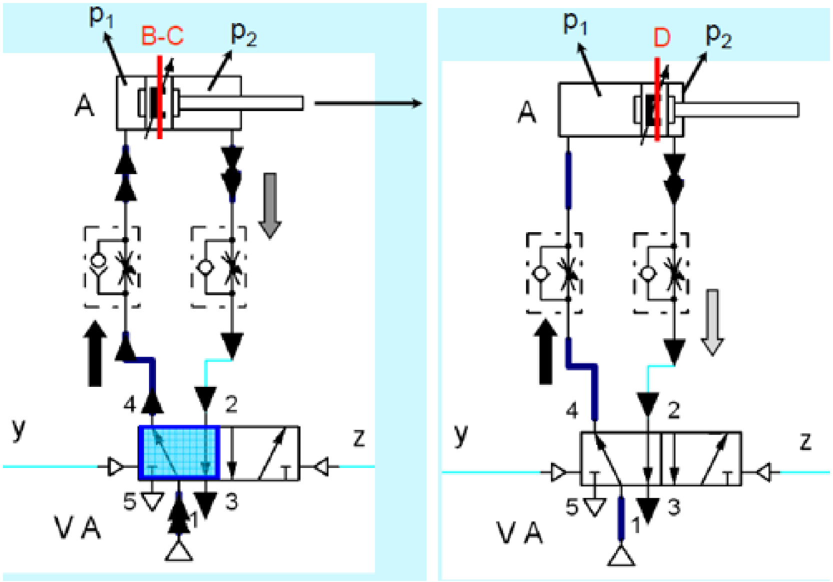


Figure 21 – first phase and steady-state

When the break-away point is overcome, the piston starts to move. First phase: initial acceleration of the piston.

When the steady-state condition is reached, the piston moves at constant velocity v.

The piston reaches the fully extended position and stops its motion (v=0).

The front chamber completes its exhausting until the ambient is reached (p2=0);

The pressure in the rear chamber increases up to the supply value (p1=ps).

PUMP

Mechanical pumps may be submerged in the fluid they are pumping or be placed external to the fluid. Pumps can be classified according to their method of displacement into positive displacement pumps, impulse pumps, velocity pumps, gravity pumps, steam pumps and valveless pumps. There are two basic types of pumps: positive displacement and centrifugal. Although axial-flow pumps are frequently classified as a separate type, they have essentially the same operating principles as centrifugal pumps.

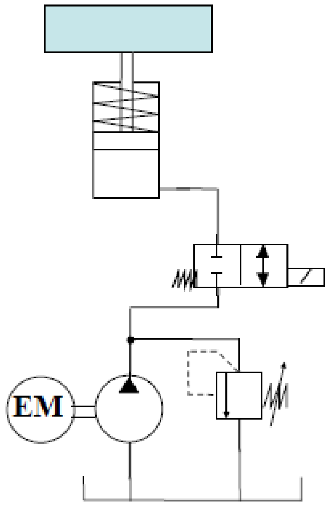
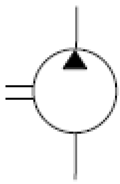


Figure 22 – example of circuit diagram

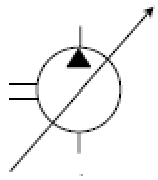
A fixed displacement pump driven by an electric motor operates a single rod cylinder. The circuit is protected against overload by a pressure relief valve. The lifting function is realized using an easy 2/2 directional control valve, which is operated by a solenoid.

1. Symbols

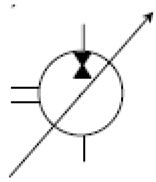
Fixed Displacement pump



Variable Displacement pump



Variable Displacement machine



2. Hydraulic pumps, calculation formulas:
   a. Flow:

$$Q = n \cdot V_{stroke} \cdot \eta_{vol}$$

   where,
   - Q, flow (m^3/s)
   - n, stroke frequency (Hz)
   - V_{stroke}, stroked volume (m^3)
   - η_{vol}, volumetric efficiency

b. Power:

$$P = \frac{n \cdot V_{stroke} \cdot \Delta p}{\eta_{mech}}$$

where,

- P, power(W)
- n, stroke frequency (Hz)
- V_{stroke}, stroked volume (m^3)
- Δp, pressure difference over pump (Pa)
- η_{mech}, mechanical efficiency

c. Mechanical Efficiency:

$$\eta_{mech} = \frac{T_{theoretical}}{T_{actual}} \cdot 100\%$$

where,

- η_{mech}, mechanical pump efficiency percent
- $T_{theoretical}$, theoretical torque to drive (N·m)
- T_{actual}, actual torque to drive (N·m)

d. Hydraulic Efficiency:

$$\eta_{hydr} = \frac{Q_{actual}}{Q_{theoretical}} \cdot 100\%$$

where,

- η_{hydr}, hydraulic pump efficiency
- $Q_{theoretical}$, theoretical flow rate output (m^3)
- Q_{actual}, actual flow rate output (m^3)

EXAMPLE

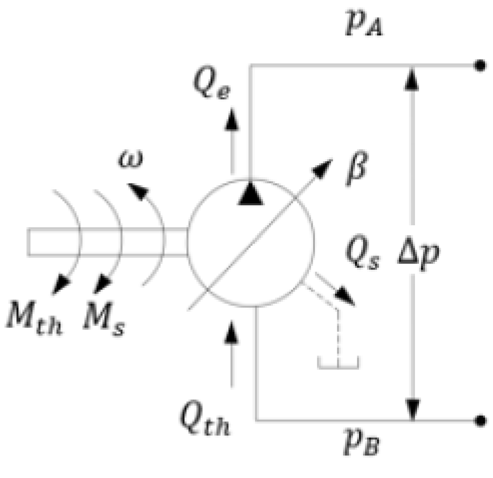


Figure 23 - Pump scheme

Pumping mode:

$$Q_e = Q_{th} - Q_s = \beta \cdot V_d \cdot \omega - Q_s$$

Where Q_e is the effective flow rate, Q_{th} is the theoretical flow rate, β is the swash plate angle, ω is the egine angular speed.

$$M_e = M_{th} + M_s = \beta \frac{\Delta p \cdot V_d}{2\pi} + M_s$$

Where M_e is the effective torque, M_{th} is the theoretical torque, M_s are the torque losses.

Useful links:

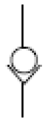
https://en.wikipedia.org/wiki/Pump#Hydraulic_ram_pumps

OTHER FUNCTIONAL ELEMENTS

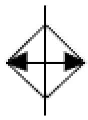
Pneumatic and hydraulic symbols conform to and are devised from the International Standard ISO 1219. This is a reference for the schematic symbols used in fluid power schematics, hydraulic schematics, pneumatic schematics, diagrams, and circuits.

Accumulator

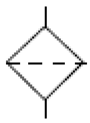
Check Valve



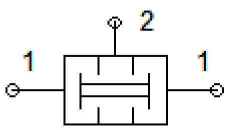
Cooler



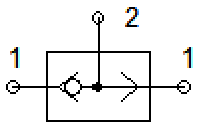
Filter



Logic Valve (And)

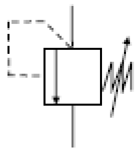


Logic Valve (Or)

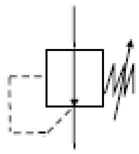


Reservoir / Tank

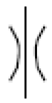
Pressure Relief Valve



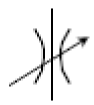
Pressure Reduction Valve



Throttling Valve

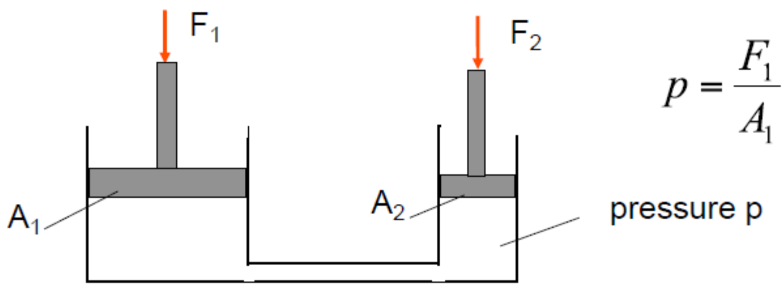


Adjustable Throttling Valve



Basic of Bernoulli

"Any change of pressure at any point of an incompressible fluid at rest is transmitted equally in all directions." Formulated 1651 by Pascal.



$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Figure 24 – Pascal law

Thus, it is possible to transmit forces using the static pressure of a fluid. The hydrostatic pressure is given by the ratio of the force acting on a fluid column and the related area.

In most flows of liquids, and of gases at low Mach number, the density of a fluid parcel can be considered to be constant, regardless of pressure variations in the flow. Therefore, the fluid can be considered to be incompressible and these flows are called incompressible flows. Bernoulli performed his experiments on liquids, so his equation in its original form is valid only for incompressible flow. A common form of Bernoulli's equation, valid at any arbitrary point along a streamline, is:

$$\frac{1}{2}\rho v^2 + \rho g h + p = \text{constant}$$

or

$$\frac{1}{2}v^2 + gh + \frac{p}{\rho} = \text{constant}$$

where:

v (m/s) is the fluid flow speed at a point on a streamline,

g (m/s^2) is the acceleration due to gravity,

h (m) is the elevation of the point above a reference plane,

p (Pa) is the pressure at the chosen point,

ρ (kg/m^3) is the density of the fluid at all points in the fluid.

EXAMPLE

In an actuator with the same pressure in the two chambers. It is known that the two piston areas are A1 and A2 respectively, the given force F1 and F2.

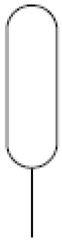
$$F_1 A_1 = F_2 A_2$$

$$\frac{F_1}{F_2} = \frac{A_2}{A_1}$$

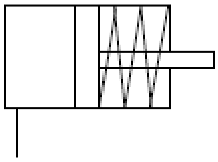
The working principle is similar to a transformer: the transformation ratio is given by the area ratio.

IV. Exercise

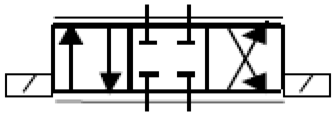
1. Describe the name and function of following pneumatic/hydraulic symbols



a.



b.

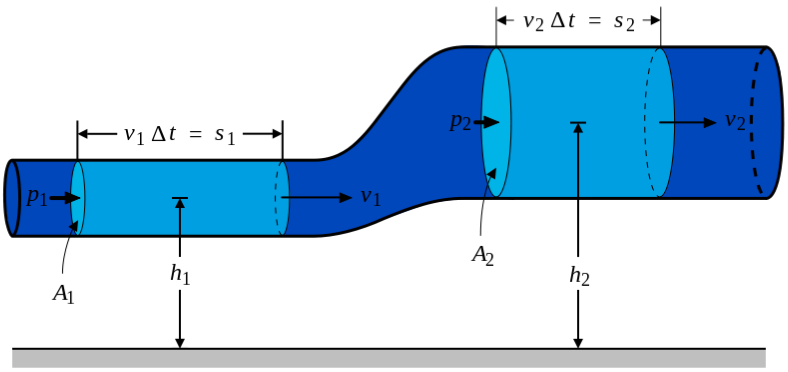


c.

Answer:

a. Accumulator: it is an energy storage device for maintaining system stability.
b. Single rod cylinder: it is a single-acting cylinder relies on the spring.
c. 4/3 directional control valve, electrically operated: it allows fluid flow into different paths from one or more sources to control different actions of the actuator.

2. Given all parameters in the following picture, this is a dynamic fluid tube. To prove the truth of Bernoulli's equation ($\frac{1}{2}\rho v^2 + \rho g h + p = constant$).



Answer:

The energy of a fluid that is driven by pressure:

$$F_1S_1 - F_2S_2 = p_1A_1v_1\Delta t - p_2A_2v_2\Delta t$$

The energy lost by the force of gravity:

$$mgh_1 - mgh_2 = \rho g A_1 v_1 h_1 \Delta t - \rho g A_2 v_2 h_2 \Delta t$$

The kinetic energy of the fluid can be obtained:

$$\frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 = \frac{1}{2}\rho A_2 v_2 \Delta t v_2{}^2 - \frac{1}{2}\rho A_1 v_1 \Delta t v_1{}^2$$

According to the law of conservation of energy, the kinetic energy of the fluid would be the sum of energy of fluid given by driven pressure and energy lost by the force of gravity.

$$\frac{1}{2}\rho A_2 v_2 \Delta t v_2{}^2 - \frac{1}{2}\rho A_1 v_1 \Delta t v_1{}^2 = p_1 A_1 v_1 \Delta t - p_2 A_2 v_2 \Delta t + \rho g A_1 v_1 h_1 \Delta t - \rho g A_2 v_2 h_2 \Delta t$$

Owing to continuity equation (equal discharge):

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = constant$$

the previous equation would become:

$$\frac{1}{2}\rho v^2 + \rho g h + p = constant$$

or:

$$\frac{1}{2}v^2 + gh + \frac{p}{\rho} = \text{constant}$$

Review Exercise

1. Please find dissipative energy of damper B_{r1} in figure a system, where θ_1 and θ_2 are turning angle of turn-plate, B_{r1} and B_{r2} is the damping coefficient of the damper.

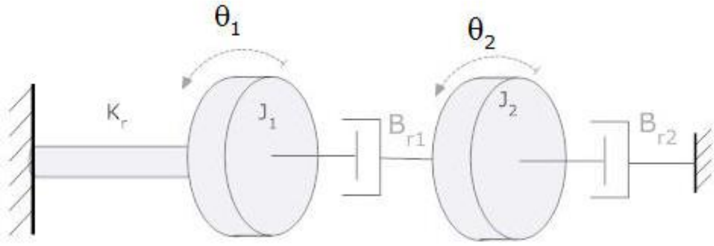


Figure a

2. Please find all dissipative energy equations in figure b system.

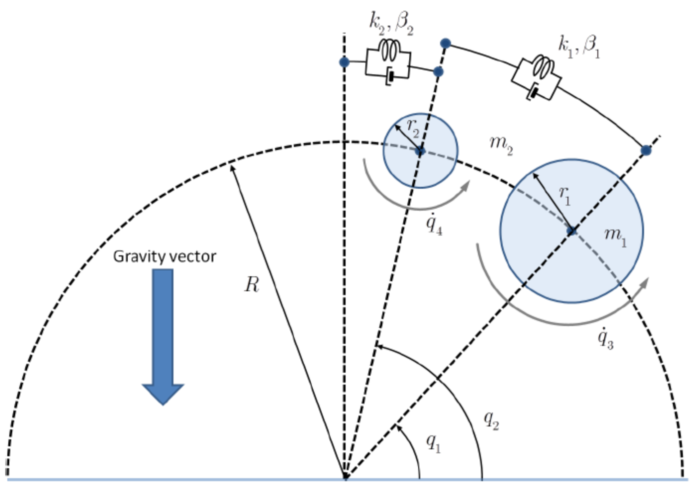


Figure b

3. Which is the physical meaning of viscosity and what is the effect of viscosity? According to figure c, please express its continuity equation and compute its discharge.

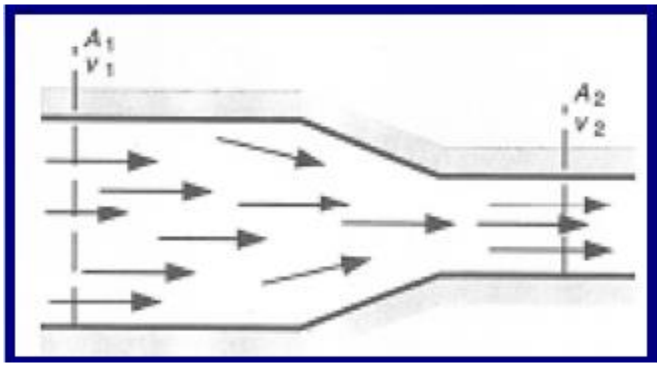


Figure c

4. What is the working difference of following two cylinders?