

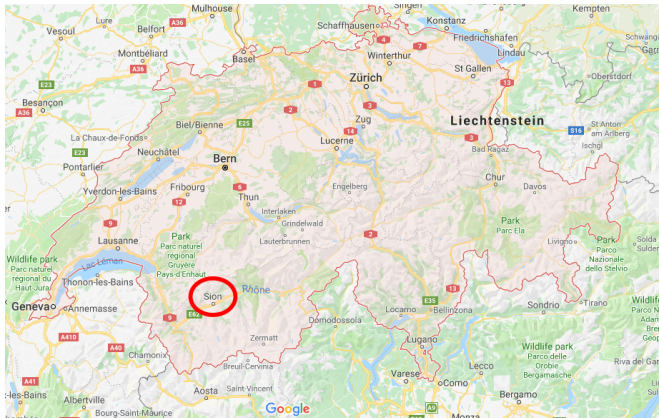
Robustness of elections results against external influence

Robin Delabays

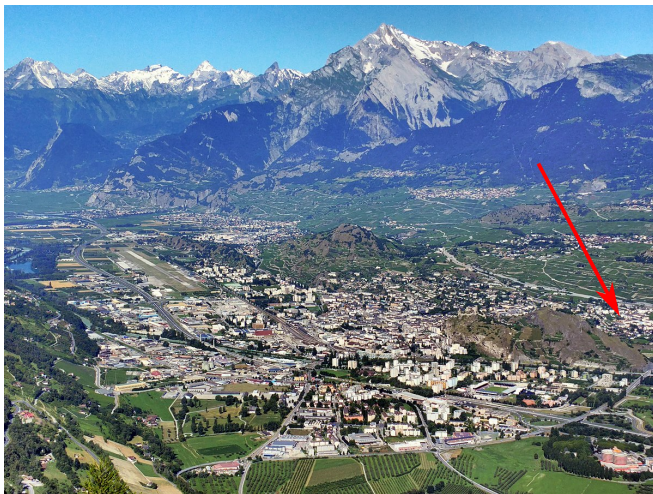
robin.delabays@hevs.ch

joint work with G. M. Givi and P. Jacquod

Where is HES-SO?



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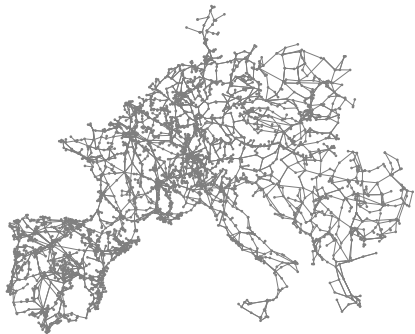
Motivation

Electrical networks:

- ▶ Dynamical systems:

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = \omega_i - \sum_j B_{ij} \sin(\theta_i - \theta_j),$$

- ▶ Complex networks:



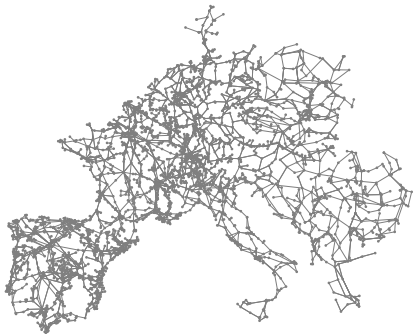
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**Where else can we
apply our results?**

The model

Taylor model (continuous time French-DeGroot):

$$\dot{x}_i = \sum_j w_{ij}(x_j - x_i) + \alpha_i(x_i^{(0)} + \omega_i - x_i),$$

- ▶ x_i : i 's opinion;
- ▶ w_{ij} : weight accorded to j by agent i ;
- ▶ $x_i^{(0)}$: i 's natural opinion;
- ▶ ω_i : external influence on i ;
- ▶ α_i : attachment of i to their natural opinion.

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Consider $w_{ij} = a_{ij}$, $\alpha_i = 1$, and $\omega_i \in \{-1, 0, 1\}$.

The model

$$\dot{\mathbf{x}} = -(\mathbb{L} + \mathbb{I})\mathbf{x} + \mathbf{x}^0 + \boldsymbol{\omega},$$

$$\implies \mathbf{x}^* = (\mathbb{L} + \mathbb{I})^{-1}(\mathbf{x}^0 + \boldsymbol{\omega})$$

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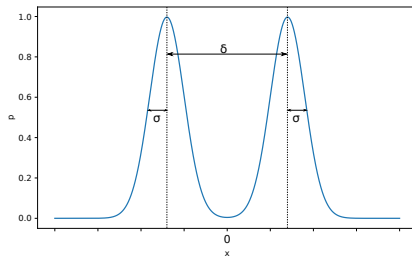
$$\implies \mathbf{x}^* = (\mathbb{L} + \mathbb{I})^{-1}(\mathbf{x}^0 + \boldsymbol{\omega})$$

Outcome:

$$o(\mathbf{x}) = \sum_i \text{sign}(x_i)$$

Assumed positive (wlog) before influence.

Natural opinions



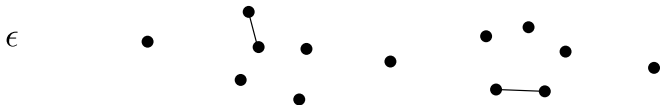
- ▶ Bimodal \implies models polarization;
- ▶ Symmetric \implies close to parity;
- ▶ Random \implies avoids degeneracies.

The network

$$(\mathbb{L}_\epsilon)_{ij} = \begin{cases} -1, & i \neq j, |x_i^{(0)} - x_j^{(0)}| \leq \epsilon, \\ 0, & i \neq j, |x_i^{(0)} - x_j^{(0)}| > \epsilon, \\ -\sum_{k \neq j} (\mathbb{L}_\epsilon)_{ik}, & i = j. \end{cases}$$

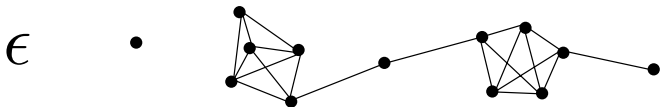
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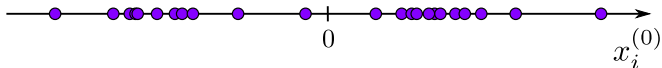
Influence strategies

Goal: Change the outcome by influencing the agents, minimizing the effort, i.e., $\|\omega\|_1$.

Question: Who should we target?

Proposed strategies:

- ▶ Random
- ▶ Minimum



- ▶ Fiedler ...

Fiedler strategy

Maximize the impact of ω on the outcome, "i.e.", on \mathbf{x}^* .

$$\delta \mathbf{x} := \mathbf{x}^*(\omega) - \mathbf{x}^*(\mathbf{0}) = (\mathbb{L} + \mathbb{I})^{-1} (\mathbf{x}^0 + \omega - \mathbf{x}^0)$$

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Decompose on the Laplacian's eigenbasis:

$$0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n, \quad \mathbf{u}^{(1)} = n^{-1/2}(1, \dots, 1),$$

$$\delta \mathbf{x} = \sum_i \frac{\omega^\top \mathbf{u}^{(i)}}{\lambda_i + 1} \mathbf{u}^{(i)}.$$

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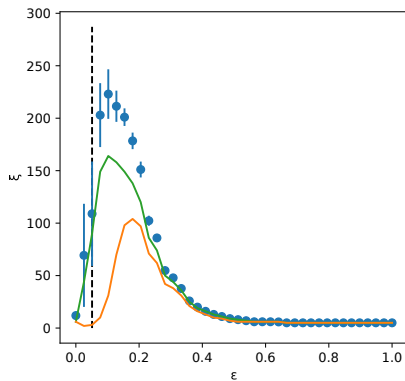
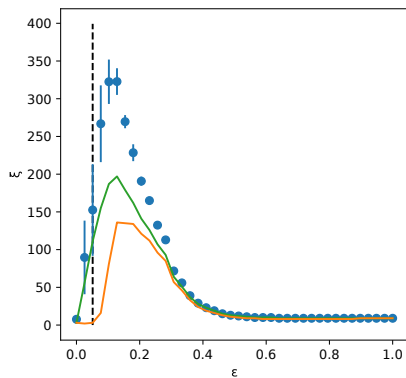
$$\delta \mathbf{x} = \sum_i \frac{\omega^\top \mathbf{u}^{(i)}}{\lambda_i + 1} \mathbf{u}^{(i)}.$$

\implies try to align ω with the *slow modes* of \mathbb{L} (but not $\mathbf{u}^{(1)}$),

\iff influence agents with large *resistive centrality*.

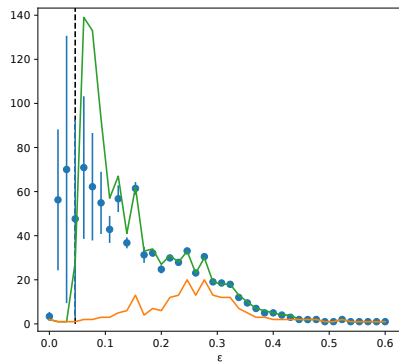
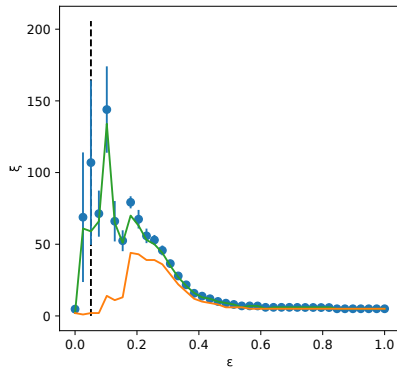
Results

Random, Minimum, Fiedler



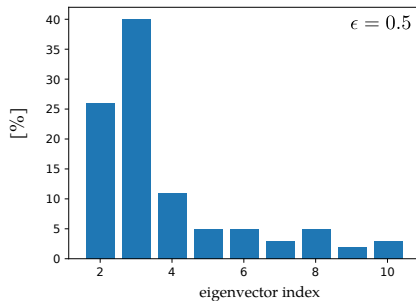
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Further questions (and conclusion)

To do:

- ▶ Desintricate the "Fiedler strategy";
- ▶ Determine the role of parameters:

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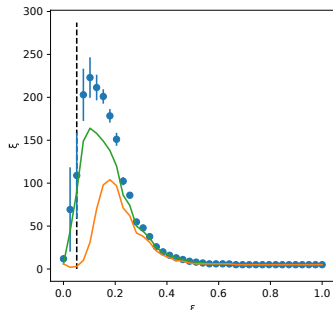
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Observation: Intermediate interaction implies the most robust opinions.



Geometry of Complex Webs 2020

GeoCoW

February 2-5, 2020

Les Diablerets
Switzerland

Organizers:

Robin Delabays (HES-SO Valais and ETH Zurich)
Matthieu Jacquemet (HES-SO Valais and Uni Fribourg)
Christian Mazza (Uni Fribourg)

Minicourse by Michael Bronstein:

"Deep Learning on Graphs and Manifolds"

Exploratory Workshop Speakers:

Michael Bronstein (Imperial College)
Moon Duchin* (Tufts)
Elisenda Feliu (Copenhagen)
Kathryn Hess-Bellwald (EPFL)
Philippe Jacquod (HES-SO Valais)
Ioan Manolescu (Fribourg)
Toshiyuki Nakagaki (Hokkaido)
Alan Newell (Tucson)
Gerd Schröder-Turk (Murdoch Perth)

* to be confirmed

sites.google.com/view/geocow2020



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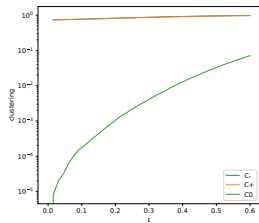
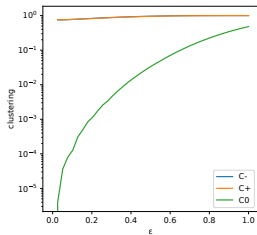
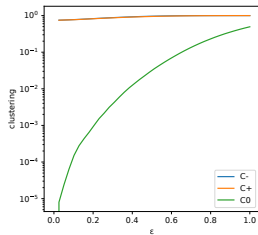


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Clusterings



Resistive centrality

Resistance distance:

$$\Omega_{ij} = \mathbb{L}_{ii} + \mathbb{L}_{jj} - \mathbb{L}_{ij} - \mathbb{L}_{ji}.$$

Centrality:

$$C(k) = \left[n^{-1} \sum_j \Omega_{kj} \right]^{-1} = \left[\sum_{\ell \geq 2} \frac{(u_k^\ell)^2}{\lambda_\ell} + n^{-2} Kf_1 \right]^{-1}$$

Kirchhoff index:

$$Kf_1 = \sum_{i < j} \Omega_{ij} = n \sum_{\ell \geq 2} \lambda_\ell^{-1}.$$