Campo di pressione medio e perturbato nello strato limite tridimensionale

Perturbative and mean pressure field in the three-dimensional boundary layer

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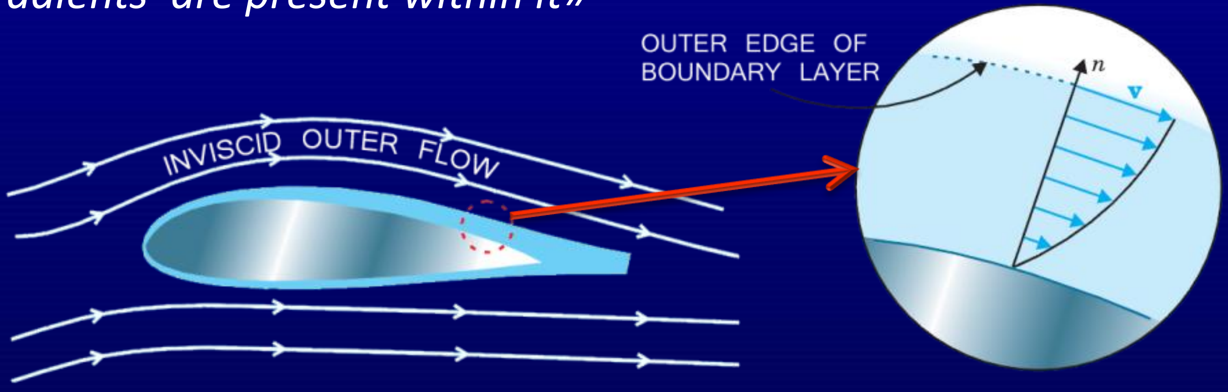
The boundary layer concept

1) Ludwig Prandtl's definition on *Third Mathematics Congress*, Heidelberg, 8 August 1904:

«*The boundary layer is a thin region near the surface, where effect of friction causes the fluid to stick to the surface (no-slip condition).*
Ouside the boundary layer the flow is essentially inviscid, while very large velocity and pressure gradients are present within it»



2) Simplification for the boundary layer of:

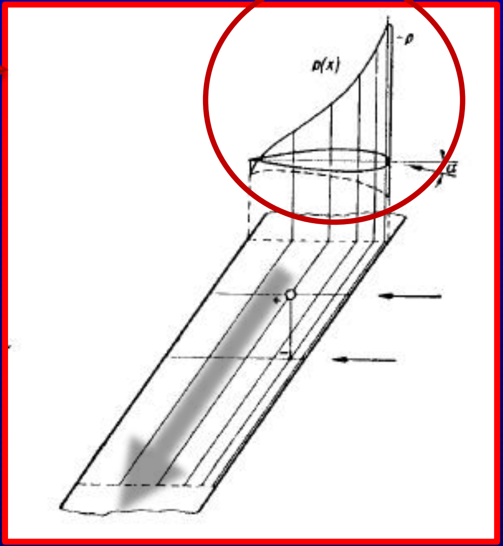
- Euler equations → Coupled non-linear partial differential equations
- Navier–Stokes equations → Viscosity effects (elliptic behavior)

Obtaining: **BOUNDARY LAYER EQUATIONS (parabolic behavior)**

Three-dimensional boundary layer

- Generation and development:

Strong pressure gradient at tip

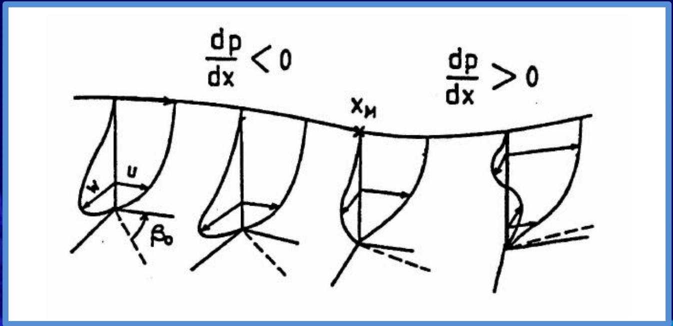


«Crossflow» Phenomenon:

Motion of fluid particles toward the receding tip

PREMATURE SEPARATION

- Velocity profiles:



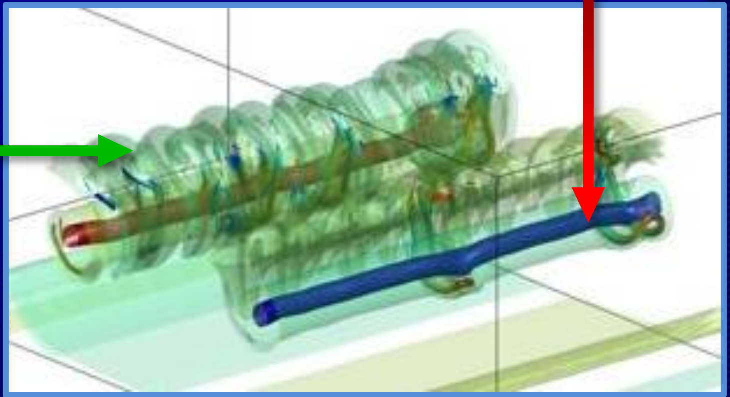
«Crossflow» velocity profile w presents:

Inflection point leads to INSTABILITY

Crossflow instability

- **PRIMARY** *receptivity process* →
  - **TRAVELLING waves**
  - **STATIONARY waves**

1) CROSSFLOW VORTICES



For very strong inflectional velocity profiles:

- **SECONDARY** → **2) SECONDARY VORTICES**
- **FINAL BREAKDOWN** (TERTIARY VORTICES)

Theory of stability

- **Method of small disturbances:**

$$u = U + \tilde{u} \quad v = V + \tilde{v} \quad w = W + \tilde{w} \quad p = P + \tilde{p}$$

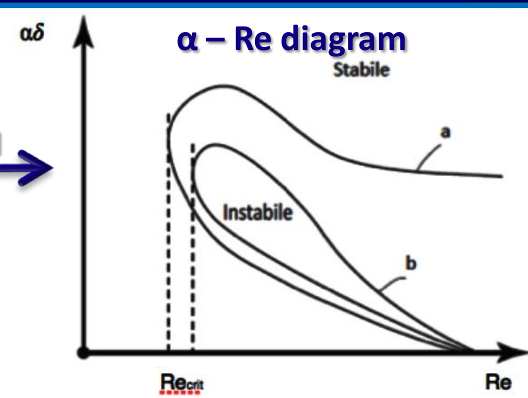
- **Orr-Sommerfeld equation:**

$$(U - c)(\phi'' - \alpha^2\phi) - U''\phi = -\frac{i}{\alpha R}(\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi)$$

EIGENVALUE PROBLEM →

$$c = c_r + ic_i$$

$$\begin{cases} c_i > 0 \\ c_i < 0 \end{cases}$$



Mathematical model

- **BASE FLOW**

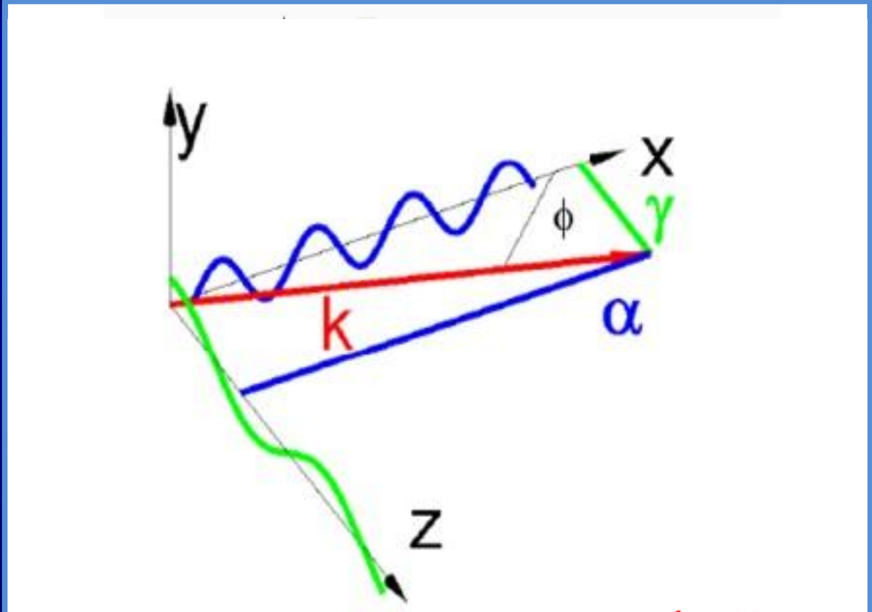
Crossflow component generates V_∞:

$$\begin{cases} U_s = U(x, Re) \\ W_s = U(x, Re) \end{cases}$$

- **Linearized PERTURBED SYSTEM**

$$\tilde{u}\ ,\ \tilde{v}\ ,\ \tilde{w} \ll U_\infty\ , U_\infty$$

$$k = \sqrt{\alpha_r{}^2 + \gamma^2} \longrightarrow \lambda = \frac{2\pi}{k}$$



Obliquity angle: $\phi = \tan^{-1}(\gamma/\alpha_r)$

➡ made DIMENSIONLESS with:

$$\begin{cases} \delta^* = \int_0^\infty (1 - \frac{u}{U_\infty})\, dy \\ Re_{\delta^*} = \frac{U_\infty \delta^*}{\nu} \end{cases}$$

➡ NAVIER-STOKES equations (Fourier space)

⟶ Initial and boundary conditions (y-coordinate) for $\tilde{v}$, $\tilde{\omega}$

Pressure field: NUMERICAL SIMULATION

Two methods implemented by **MATLAB® software**:

- Mutual comparison and validity control
- Temporal and efficiency evaluations: **METHOD 2**

$$\begin{cases} \dfrac{\partial\varphi}{\partial x} & \longrightarrow & i\alpha\cdot\varphi \\ \dfrac{\partial\varphi}{\partial z} & \longrightarrow & i\gamma\cdot\varphi \\ \nabla^2\varphi & \longrightarrow & \left(\dfrac{\partial^2}{\partial y^2}-k^2\right)\cdot\varphi \end{cases}$$

Spatial LAPLACE-FOURIER decomposition

$$\begin{cases} i\alpha\hat{p} = -\dfrac{\partial\hat{u}}{\partial t} - i\alpha\hat{u}U_s - \hat{v}\dfrac{\partial U_s}{\partial y} - i\gamma\hat{u}W_s + \dfrac{1}{Re_{\delta*}}\left(\dfrac{\partial^2}{\partial y^2}-k^2\right)\hat{u} \\ \dfrac{\partial\hat{p}}{\partial y} = -\dfrac{\partial\hat{v}}{\partial t} - i\alpha\hat{v}U_s - i\gamma\hat{v}W_s + \dfrac{1}{Re_{\delta*}}\left(\dfrac{\partial^2}{\partial y^2}-k^2\right)\hat{v} \\ i\gamma\hat{p} = -\dfrac{\partial\hat{w}}{\partial t} - i\alpha\hat{w}U_s - \hat{v}\dfrac{\partial W_s}{\partial y} - i\gamma\hat{w}W_s + \dfrac{1}{Re_{\delta*}}\left(\dfrac{\partial^2}{\partial y^2}-k^2\right)\hat{w} \end{cases}$$

- NUMERICAL INTEGRATION by MATLAB®: $\hat{p} = \hat{p}(y,t,\alpha,\gamma) \xrightarrow{F^{-1}} \hat{p} = \hat{p}(x,y,z,t)$

Simulation results: PARAMETRIC ANALYSIS

$$p = P(x) + \tilde{p}(x,y,z,t)$$

PERTURBATIVE PRESSURE FIELD

- **Effects of crossflow CHARACTERISTIC PARAMETERS:**

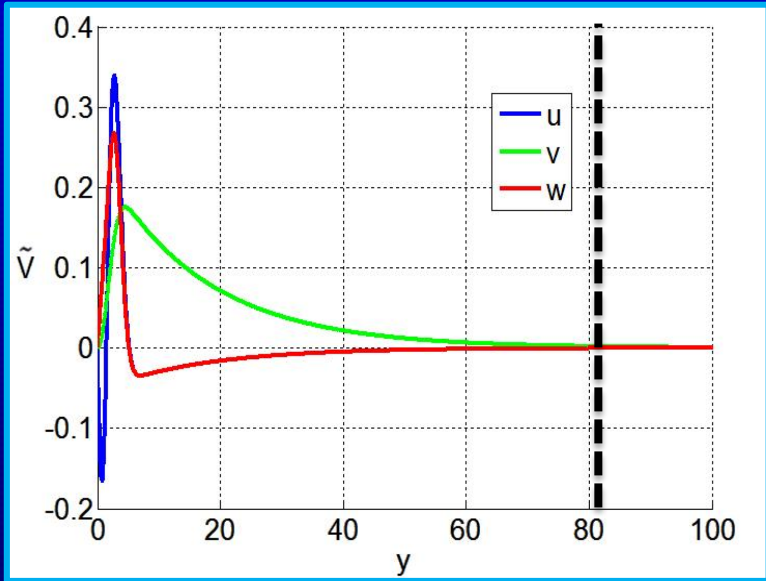
REYNOLDS NUMBER Re_{δ^*}	Re=100 , Re=5000
PRESSURE GRADIENT β	1 , -0,1988
CROSSFLOW ANGLE θ	$\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$
OBLIQUITY ANGLE ϕ	0 , $\frac{\pi}{4}$, $\frac{\pi}{2}$
WAVELENGTH NUMBER k	0.02 , 0.06 , 0.1 , 0.6 , 1 , 1.2 , 1.6 , 2

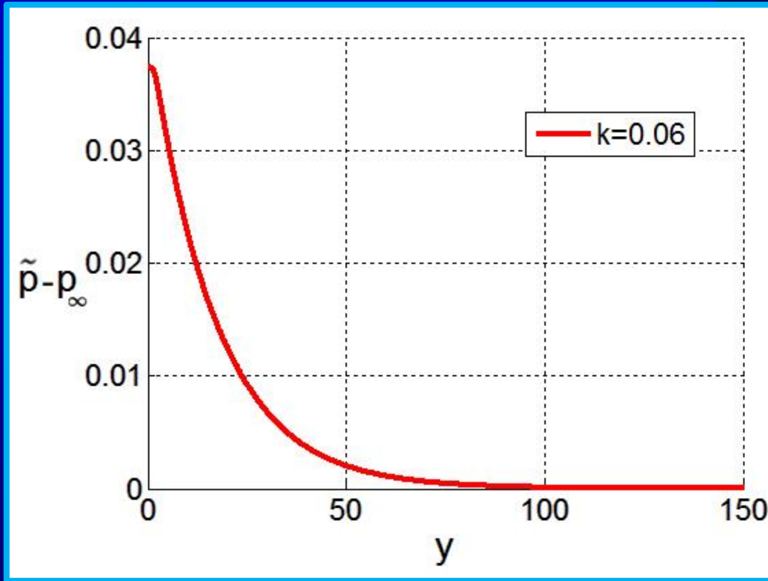
- **Comparison with AMPLIFICATION FACTOR:**

$$G(t;\alpha,\gamma) = \frac{e(t;\alpha,\gamma)}{e(t=0;\alpha,\gamma)}$$

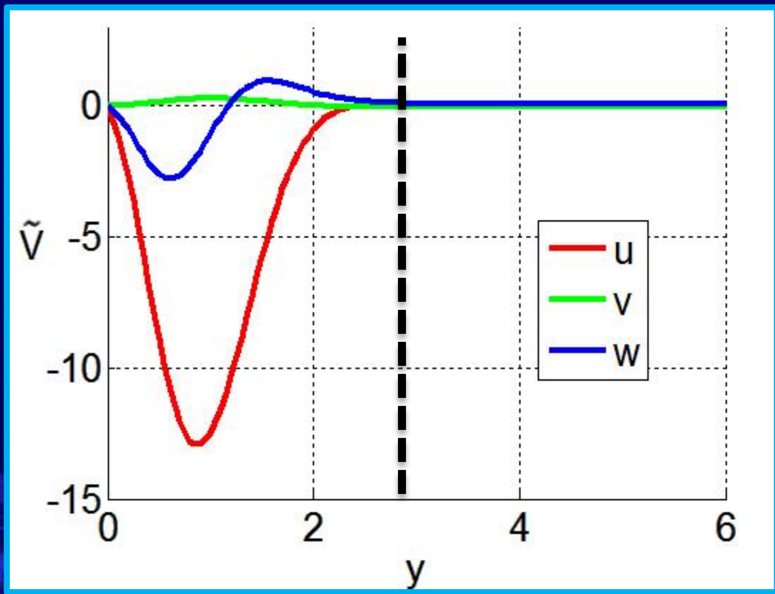
Wavenumber k

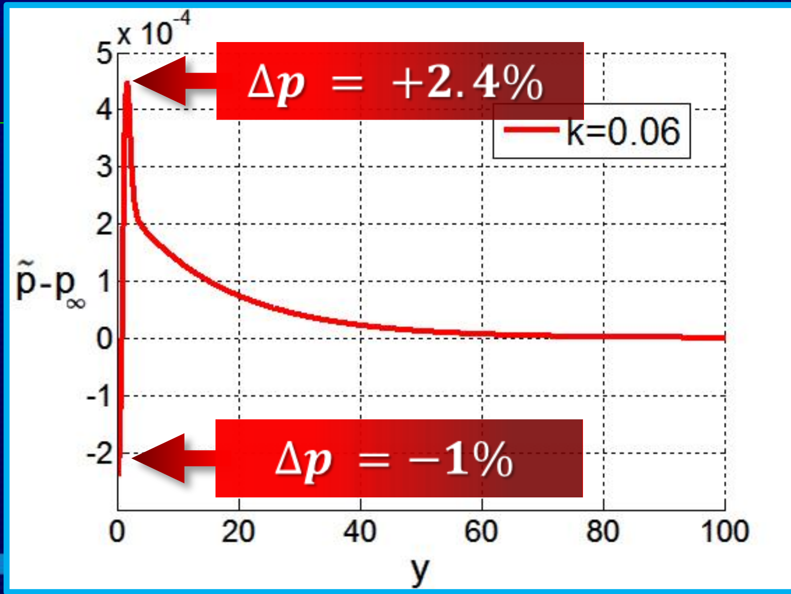
1. Flow configuration: $\theta = 30°$, $\phi = 45°$, $\beta = 1$ for $Re = 100$ → STABLE





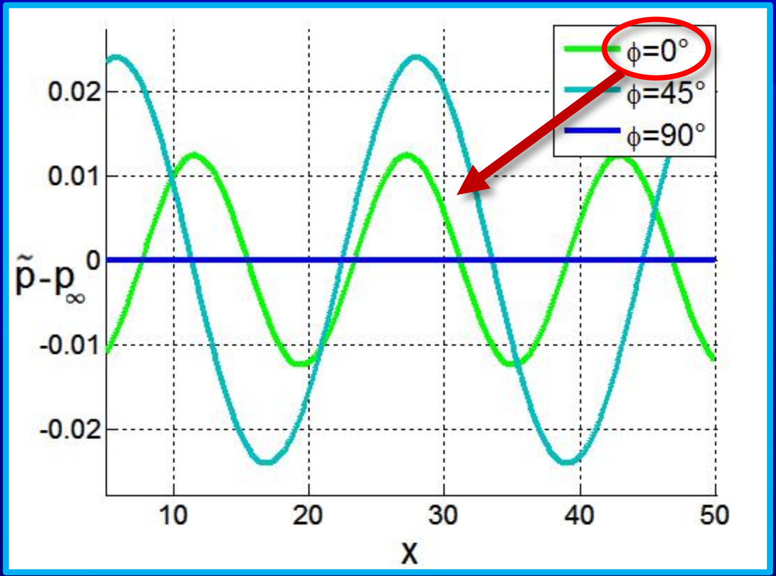
2. Flow configuration: $\theta = 60°$, $\phi = 90°$, $\beta = 1$ for $Re = 5000$ → UNSTABLE

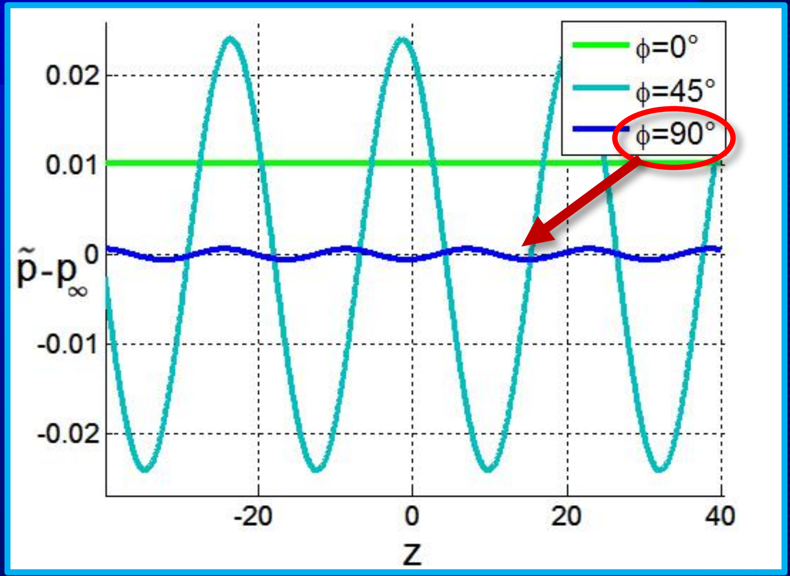




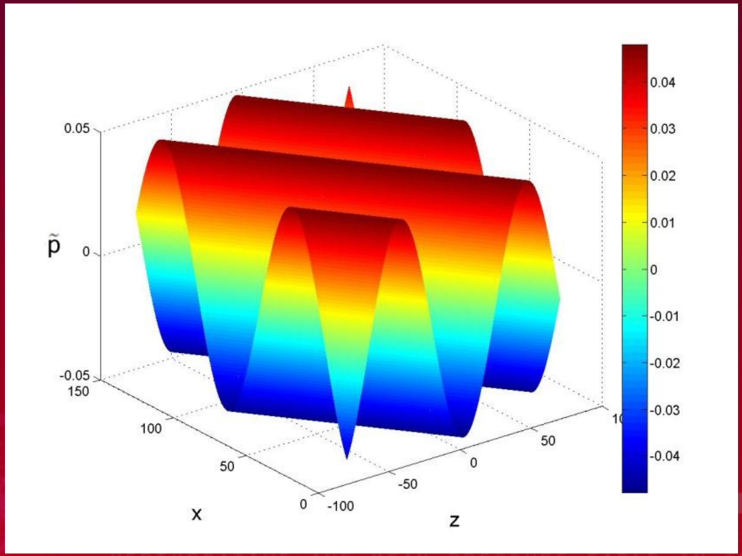
Obliquity angle ϕ

- Flow configuration: $\theta = 60°\,,\ k = 0{,}4\,,\ \beta = 1 \quad for\ Re = 5000$

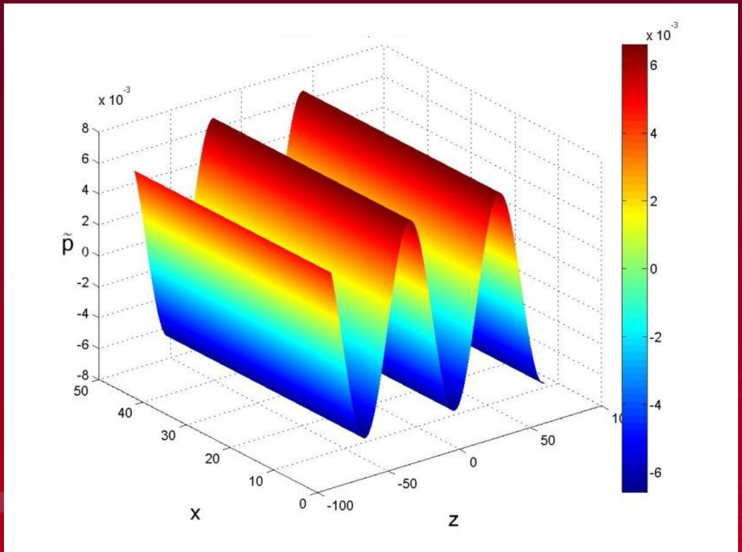




OBLIQUE WAVE for $\phi = 45°$



ORTHOGONAL WAVE for $\phi = 90°$



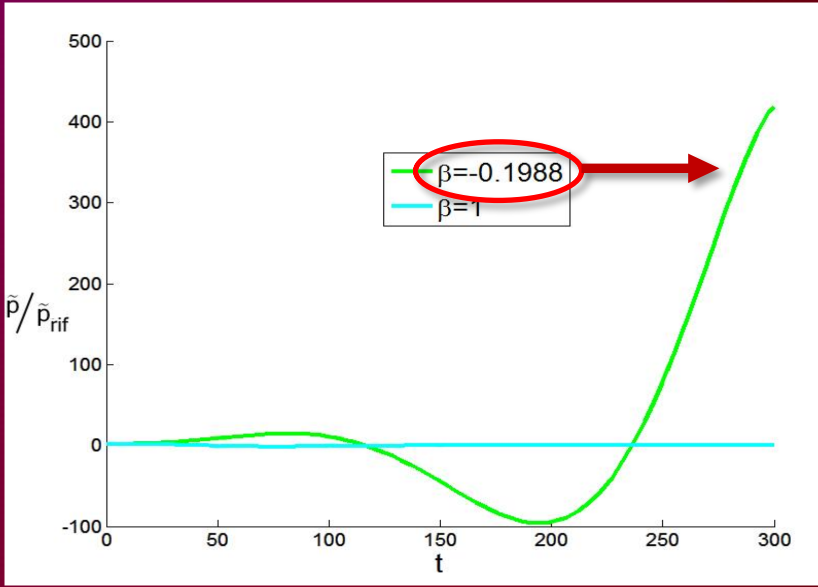
Crossflow angle θ

- **High values increase the TRANSIENT GROWTH in amplitude and time**

Flow configuration: $\phi = 45°,\ k = 0,4,\ \beta = 1 \quad for\ Re = 5000$

- **Unstable configurations:**

1. $\theta = 30°, \phi = 0° \quad for$ *Re=5000*
2. $\theta = 60°, \phi = 90° \quad for$ *Re=5000*



Pressure gradient β

$$\beta=1 \xrightarrow{\text{negative gradient}} \frac{\Delta p}{\Delta x} < 0 \xrightarrow{\text{flow acceleration}} \textbf{STABILIZING}$$

$$\beta=-0{,}1988 \xrightarrow{\text{positive gradient}} \frac{\Delta p}{\Delta x} > 0 \xrightarrow{\text{flow deceleration}} \textbf{DESTABILIZING}$$

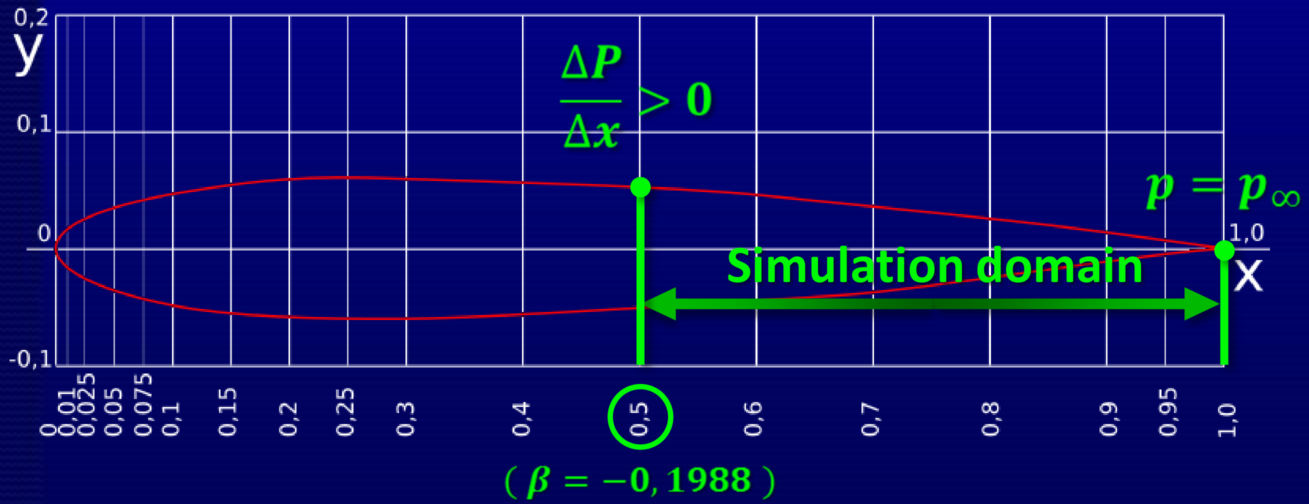
Flow configuration: $\phi = 45°,\ k = 0,4,\ \beta = 1 \quad for\ Re = 5000$

Simulation results: PRESSURE TOTAL FIELD

Effects of perturbative pressure field on mean pressure field:

$$\boldsymbol{p = P(x) + \tilde{p}(x, y, z, t)}$$

- **Simulation settings:**



$c = 1\,m$

$P_\infty = 1\,atm = 101350\,Pa$

$\rho_\infty = 1.225\,{}^{kg}/_{m^3}$

$\nu = 1.5 \times 10^{-5}\,{}^{m^2}/_{s}$

$U_\infty = 210.4626\,{}^{m}/_{s}$

$T = T_{amb}$

$Re_{\delta^*} = 5000$

$\delta(0.5) = 0.00086\,m$

$\delta^*(0.5) = 0.0002975\,m$

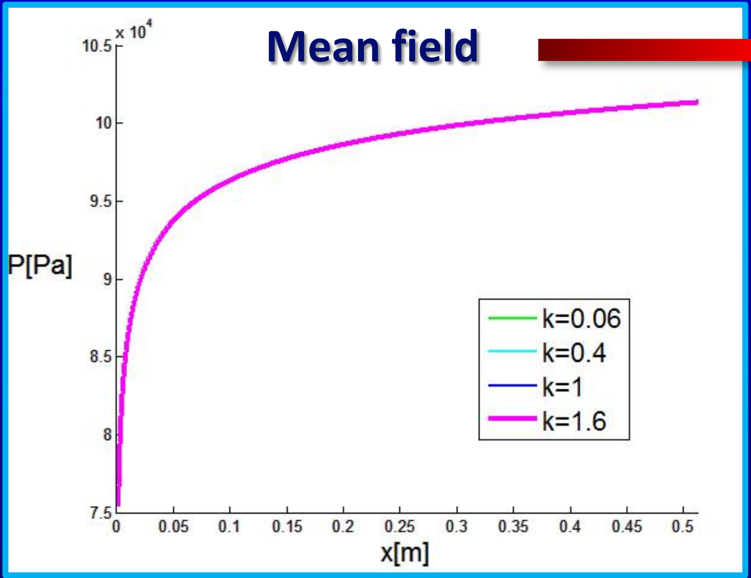
- **Mean pressure *P(x)* computation:**

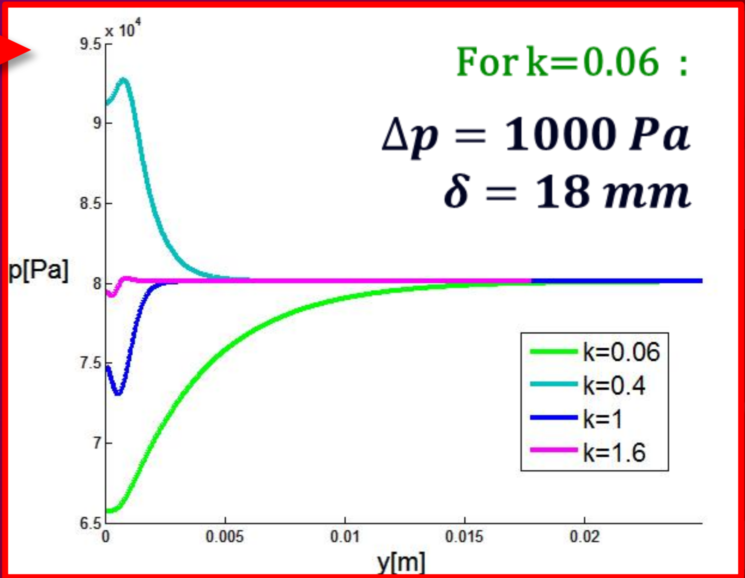
$$\begin{cases} U(x) = U_\infty x^m \\ P(x) = -\rho U_\infty x^{2m} + c \end{cases}$$

- **Quantities made dimensional with:** $\rho {U_\infty}^2, \delta, \delta^*, t_{car}$

x,y,z pressure trends

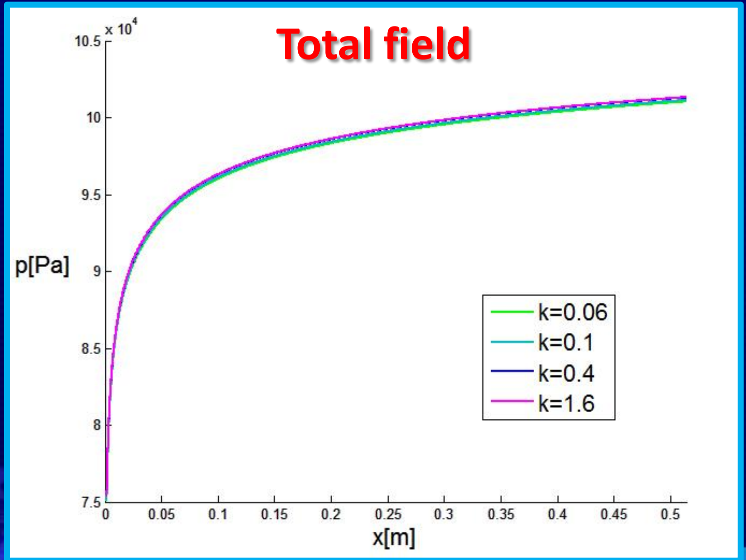
1. Flow configuration: $\theta = 30°$, $\phi = 0°$

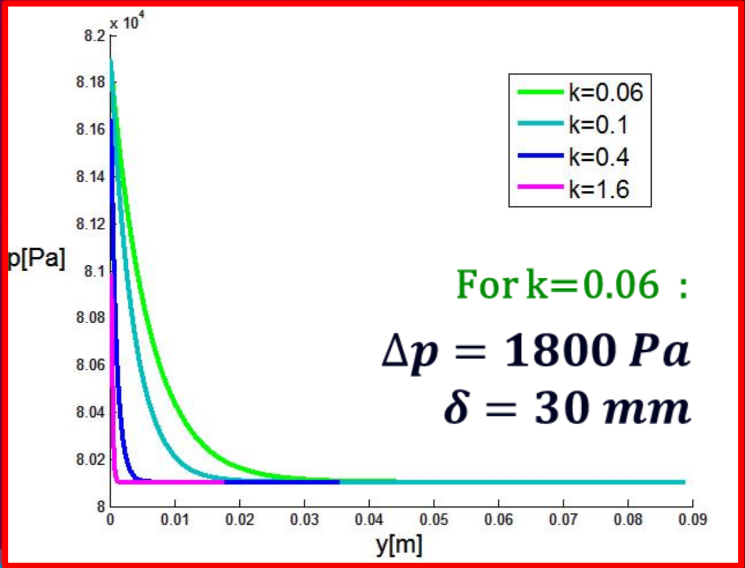




Total pressure field along y-axis

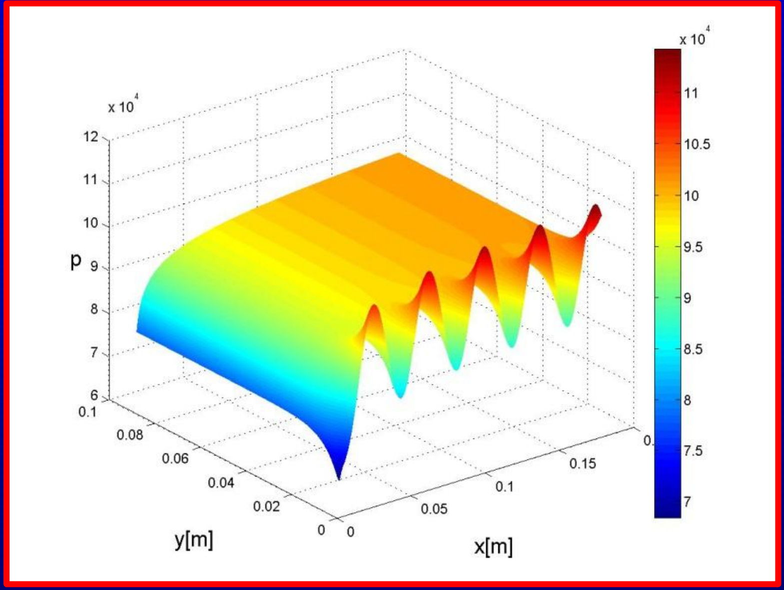
2. flow configuration: $\theta = 60°$, $\phi = 90°$

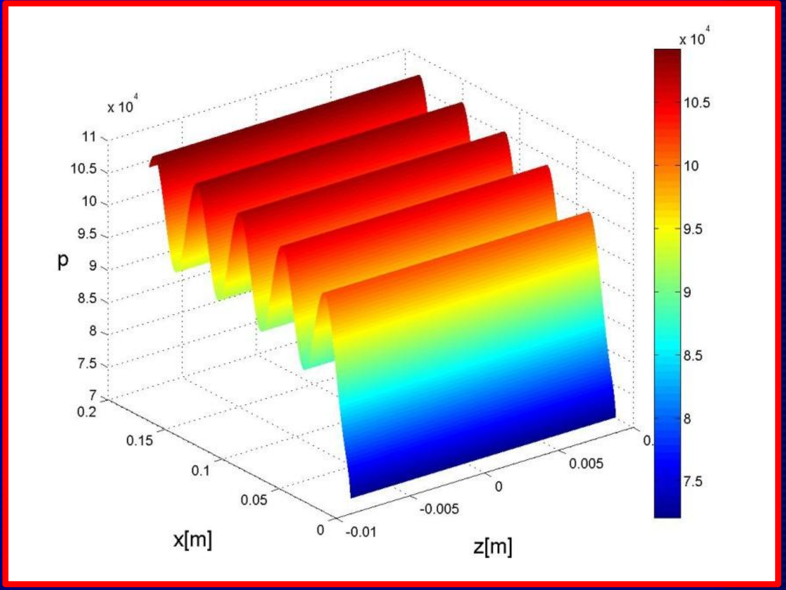




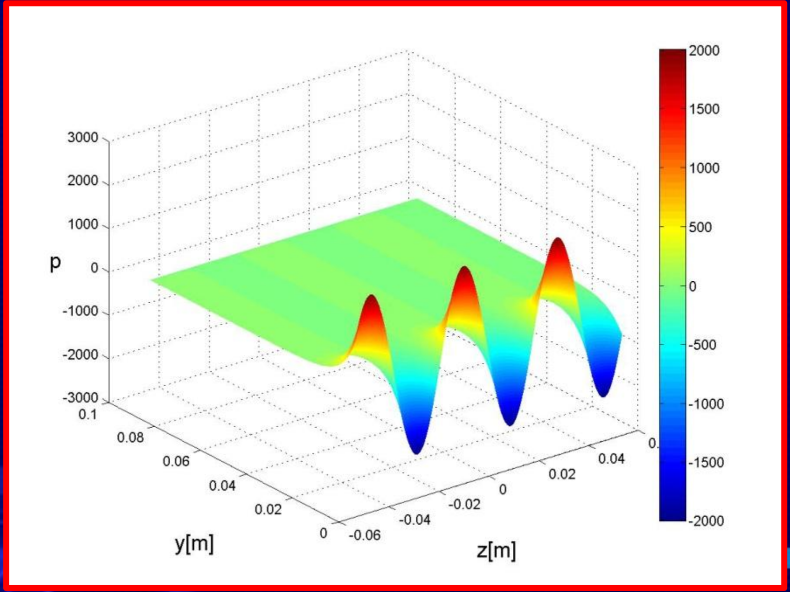
P(t) and 2D-SURFACES

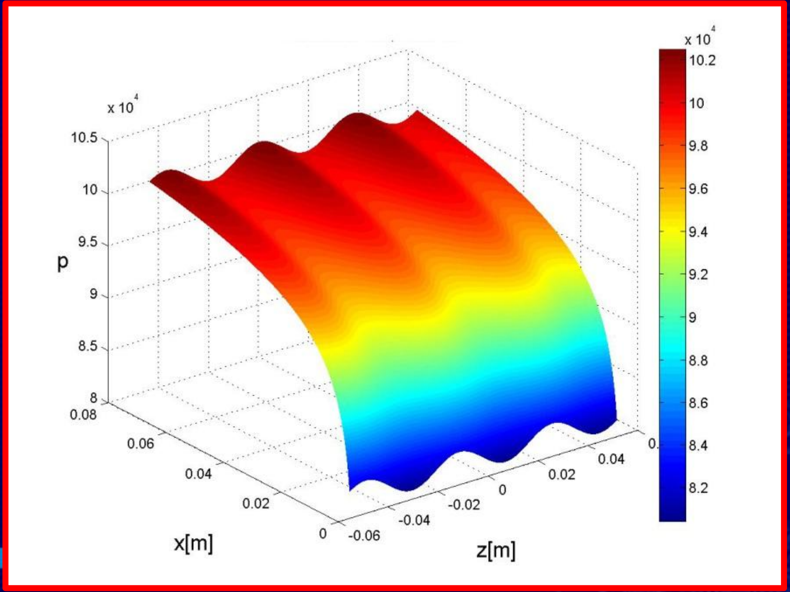
1. Flow configuration: $\theta = 30°$, $\phi = 0°$





2. flow configuration: $\theta = 60°$, $\phi = 90°$





Conclusions

1. PARAMETRIC ANALYSIS

- Correlation between G(t) and $\tilde{p}(t)$ → PRESSURE TEMPORAL GROWTH prevision
- Combination of destabilizing parameters →

$$\theta = 30°\,,\ \phi = 0°$$
$$\theta = 60°\,,\ \phi = 90°$$
$$\beta = -0,1988\ \left({}^{\Delta p}/_{\Delta x} > 0\right)$$

2. DIMENSIONAL ANALYSIS

- Strong temporal pressure growth
- Elevated spatial (x,z) and temporal oscillations

→ results for AEROELASTIC and STRUCTURAL analysis

EVENTUAL GENERATED PHENOMENA

- flattering
- vibrations

3. FURTHER DEVELOPMENT

- Increased computational domains (x,y,z,t)
- Different fly conditions included

→ $\frac{\Delta p}{\Delta x} < 0$, SECONDS , ALTITUDE

Grazie per la cortese attenzione