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POLITECNICO DI TORINO
I Facoltà di Ingegneria
Corso di Laurea Specialistica in Ingegneria Aerospaziale

A complex network approach for the analysis of turbulent flows.

Application to homogeneous isotropic turbulence.

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Turbulence

Natural phenomena:
- Atmospheric currents
- Oceanic currents
- Lava flows
- Arterial blood flows

Industrial applications:
- Aerospace
- Automotive
- Chemistry

Turbulence is a common phenomenology which is:

- Random
- Three-dimensional and multi-scale
- Irregular
- Unsteady
- Unpredictable

Turbulence studies: DNS (LES, RANS), statistical approach, energy spectrum, experimental approach

Objectives:

- Better spatial characterization
- Behavior of Taylor’s and Kolmogorov’s scales
- Complementary approach
- Gaining synthetic informations from a multipoint analysis

Turbulence is a complex system, therefore it may be studied with the complex network theory

Johns Hopkins Turbulence Databases (JHTDS)

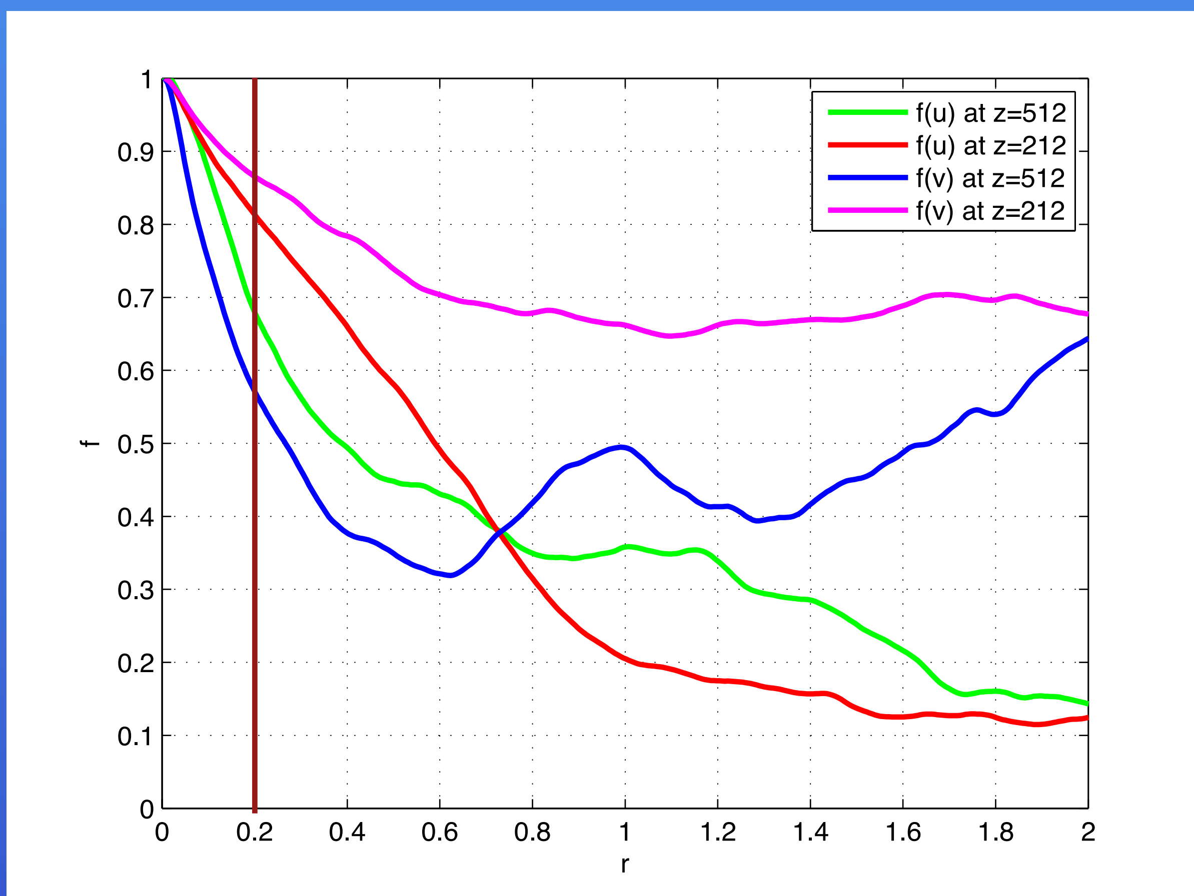
http:\\turbulence.pha.jhu.edu

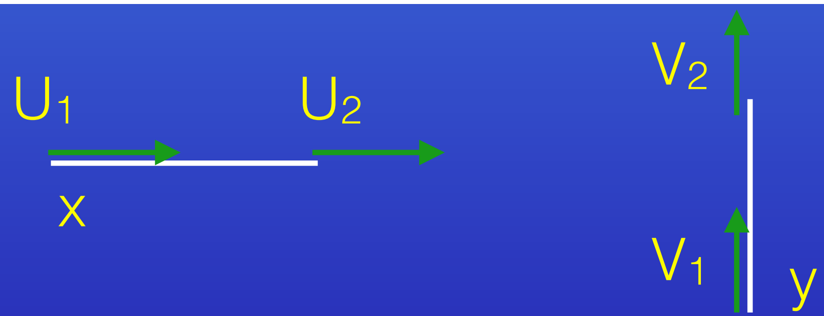
Forced isotropic turbulence:

- DNS of 1024^3 nodes, $Re_\lambda = 433$
- Energy injected by keeping constant total energy
- Storage of data after reaching stationary state
- 1024 time steps from 0 to 2.048
- Duration of stored data of about one large eddy turnover: $T_L = 2.02$
- Domain: $2\pi \times 2\pi \times 2\pi$
- Kolmogorov scale $\eta = 0.00287$
- Inserire scala Taylor $\lambda = 0.118$
- Integral scale $L = 1.376$

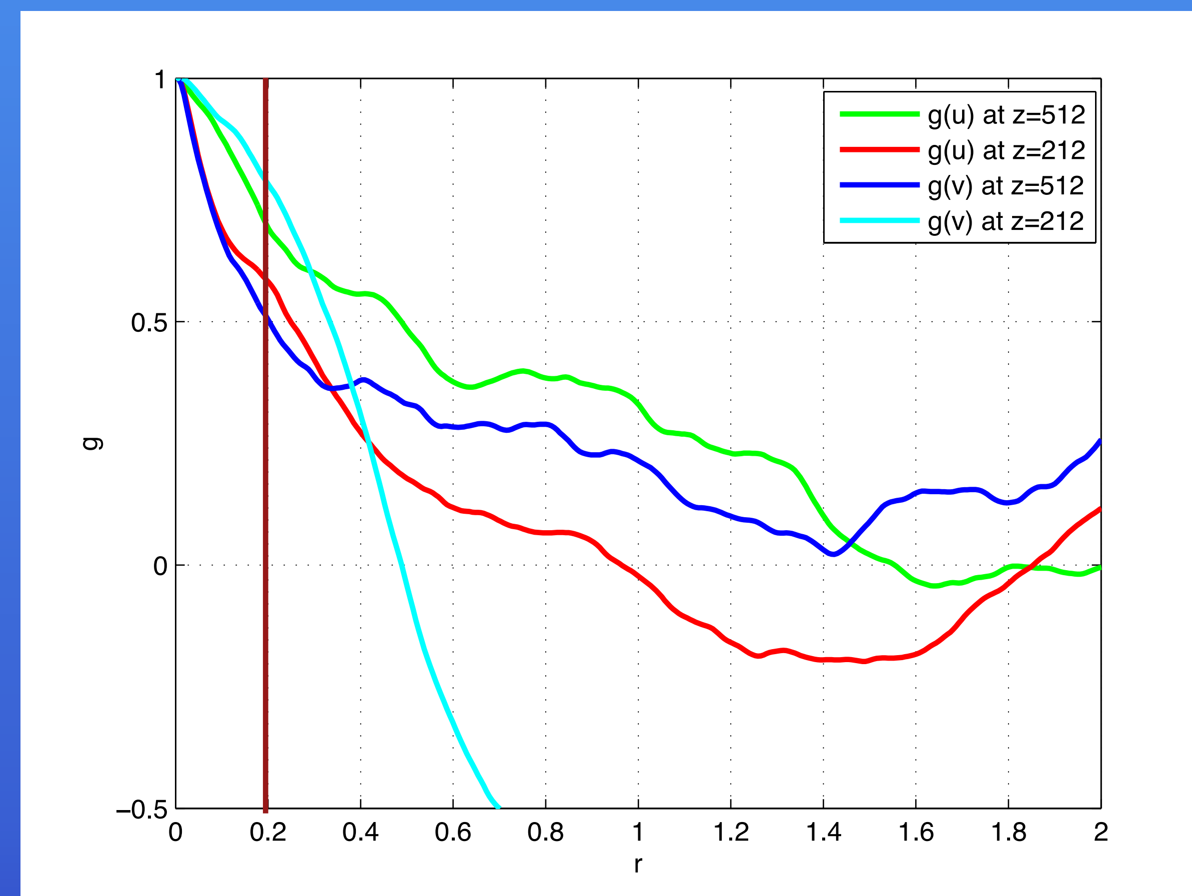
Validation of the dataset: isotropic scaling

- Longitudinal autocorrelation function





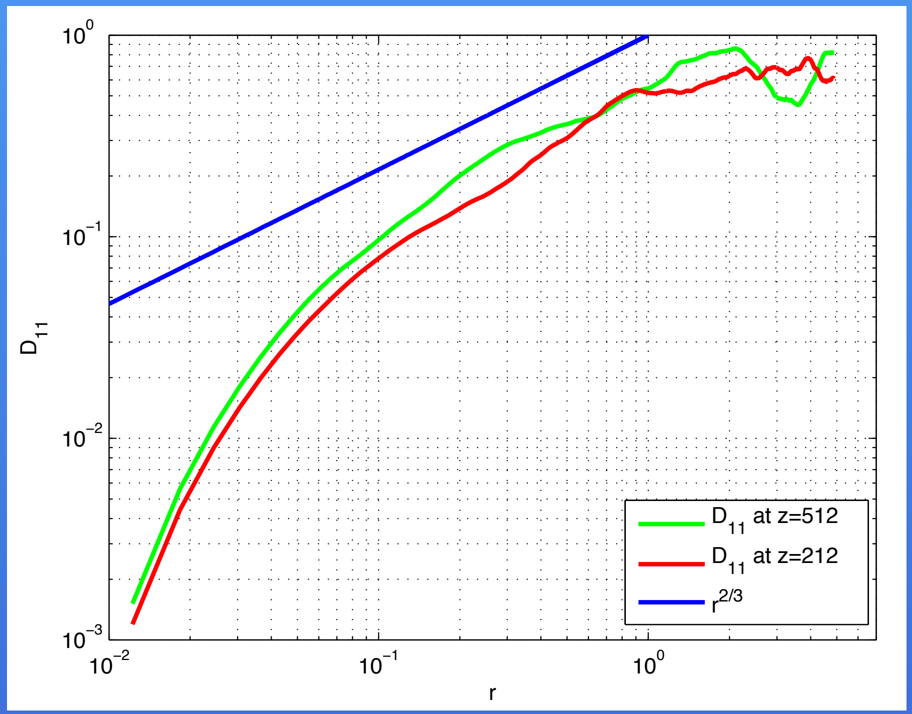
- Transverse autocorrelation functions



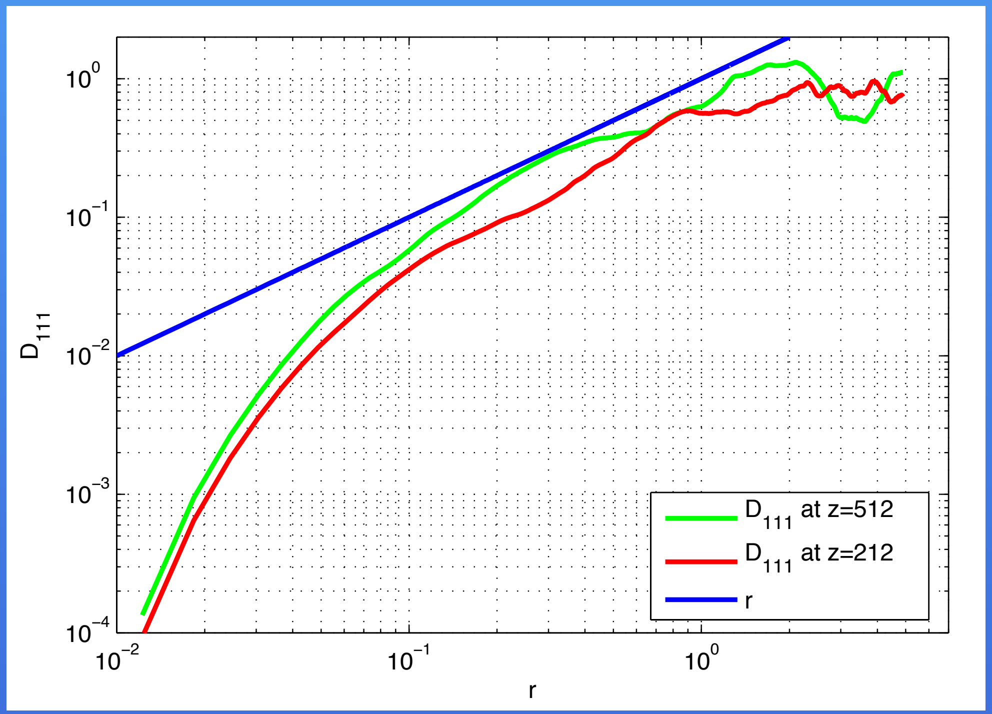
- Isotropy at small scales
- Divergence for bigger r (a bigger number of sample may be used)

A complex network approach for the analysis of turbulent flows. Application to homogeneous isotropic turbulence

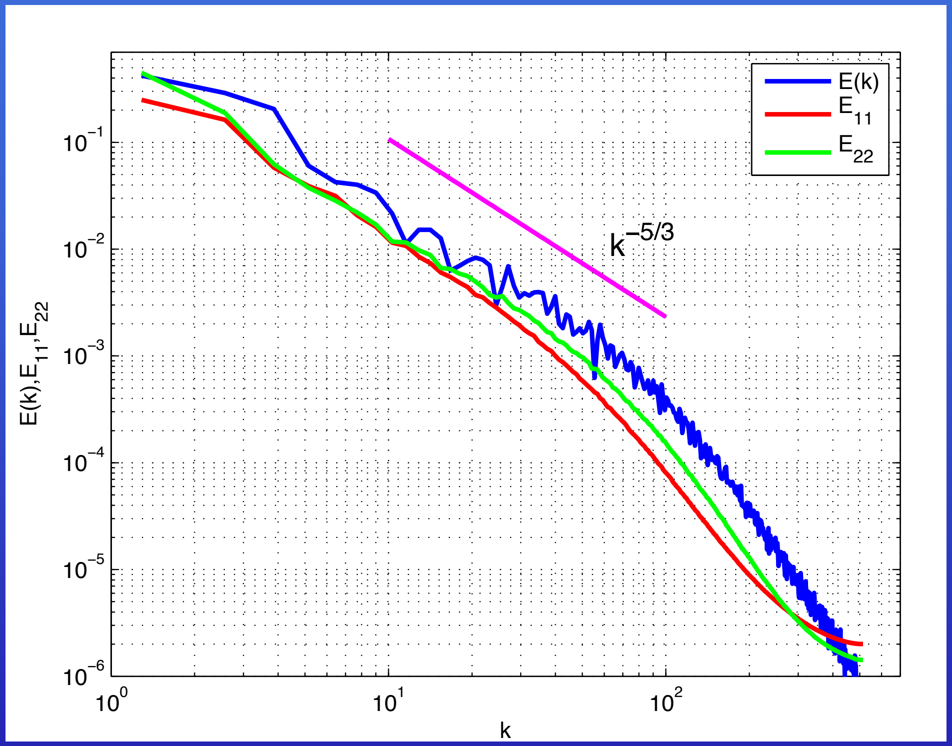
- Second-order structure functions



- Third-order structure functions



- Spectral analysis (1D, 3D)



Graph theory → Complex network theory ← Statistic physics

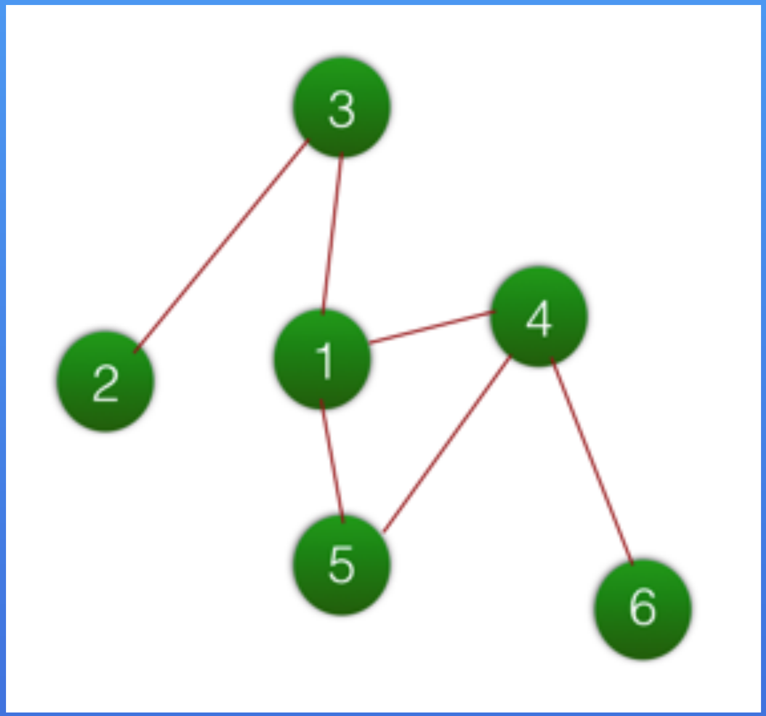
Complex network theory

- 20th century
- Still developing at fast pace
- Contributing to different fields of study
- Gaining contribution from different fields of study

Sociology (Milgram's 6 degrees of separation), Biology, Medicine (Cancer's spreading), Communications (WWW), Economy, Climatology, Earth Science (Earthquakes), Engineering (Transports)

Nodes and links form a network

$$a_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are linked} \\ 0 & \text{if } i \text{ and } j \text{ are not linked} \end{cases}$$



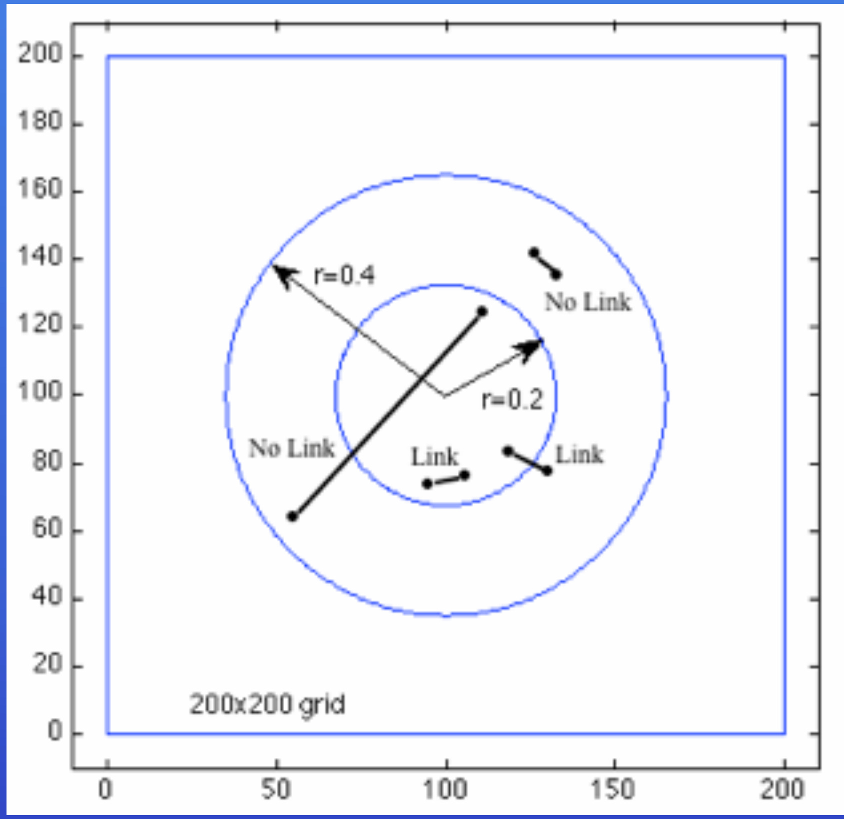
- Degree centrality $$k_i = \frac{\sum_{j=1}^{N} a_{ij}}{N-1}$$
- Weighted average topological distance $$\overline{D}_i = \frac{\sum_{j \in nn(i)} d_{ij}}{N_{ci}} \frac{N-1}{N_{ci}}$$
- Betweenness centrality $$BC_k = \sum_{i,j \neq k} \frac{\sigma_{ij}(k)}{\sigma_{ij}}$$
- Local clustering coefficient $$C_i = \frac{e(\Gamma_i)}{\frac{k_i(k_i-1)}{2}}$$

Building the network

- Two 800x800x1 grids (z=212,512)
- 160 circumferences, r = 0.4, 13333 nodes
- Temporal correlation R_{ij} between nodes of every circumference

- Threshold value $\tau = 0.5, 0.9$
- A link between nodes i and j occur if:

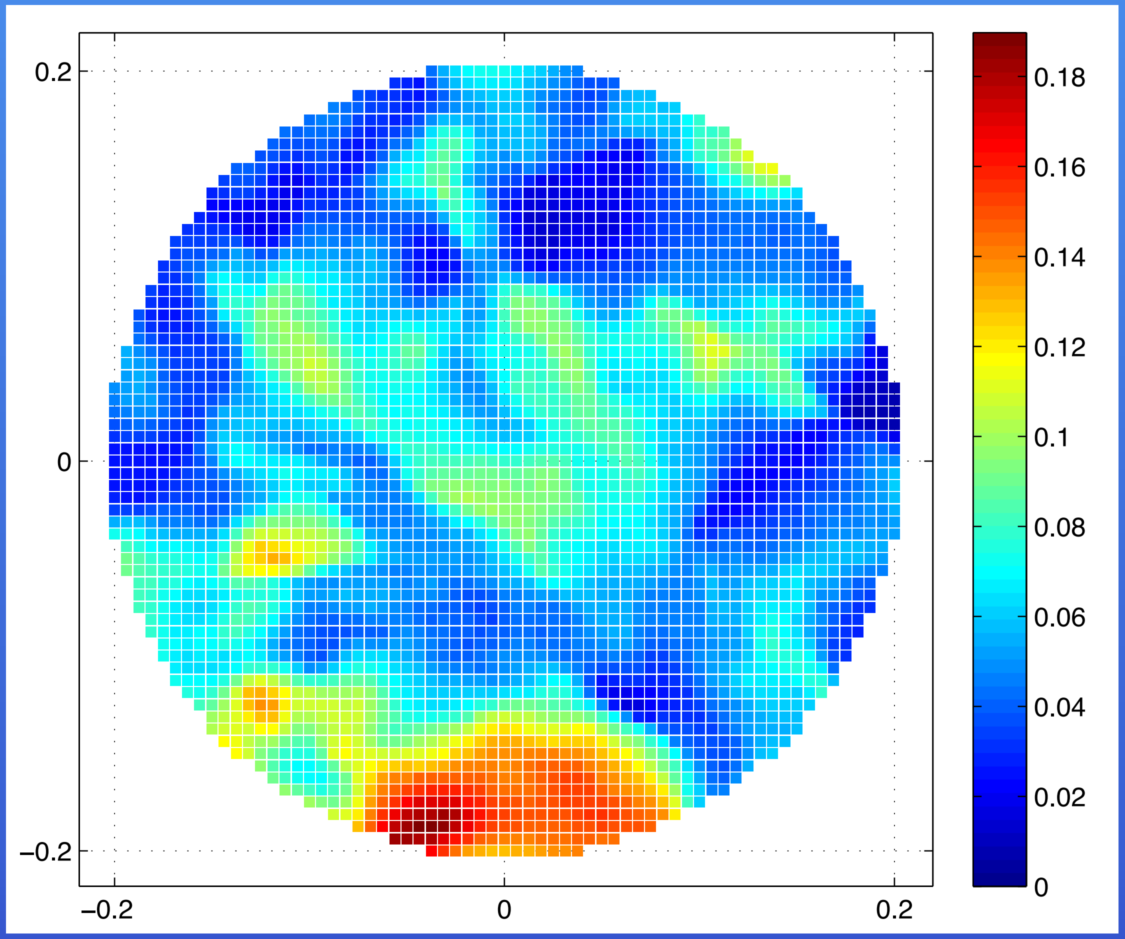
- $R_{ij} > \tau$
- At least one between nodes i and j lies inside the circumference with ray r=0.2
- The physical distance between nodes i and j is less or equal to 0.2



- Same potential number of links for every node
- Focus on a circumference of ray r = 0.2, doubling the Taylor’s scale

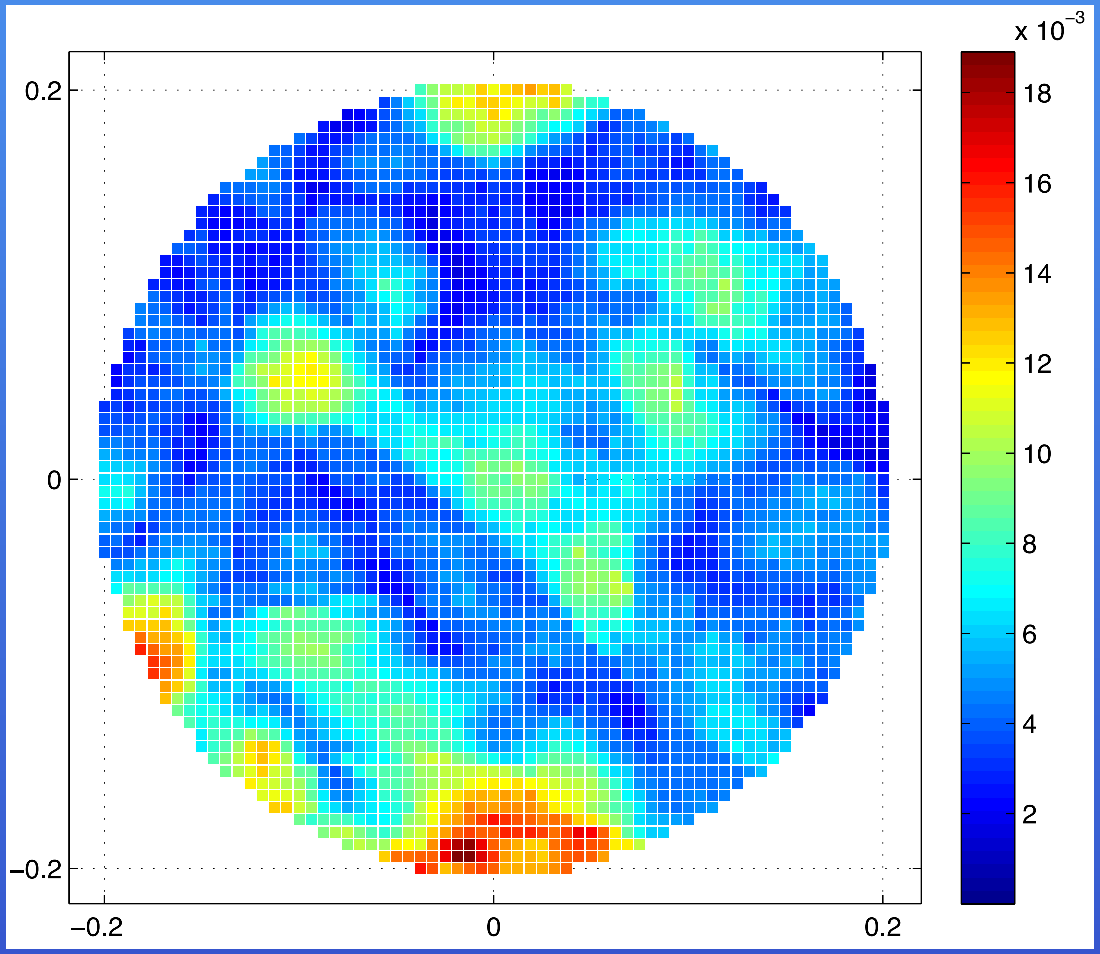
Circumference (200,600) at z=512

τ=0.5



9378 nodes, 1313722 links

τ=0.9

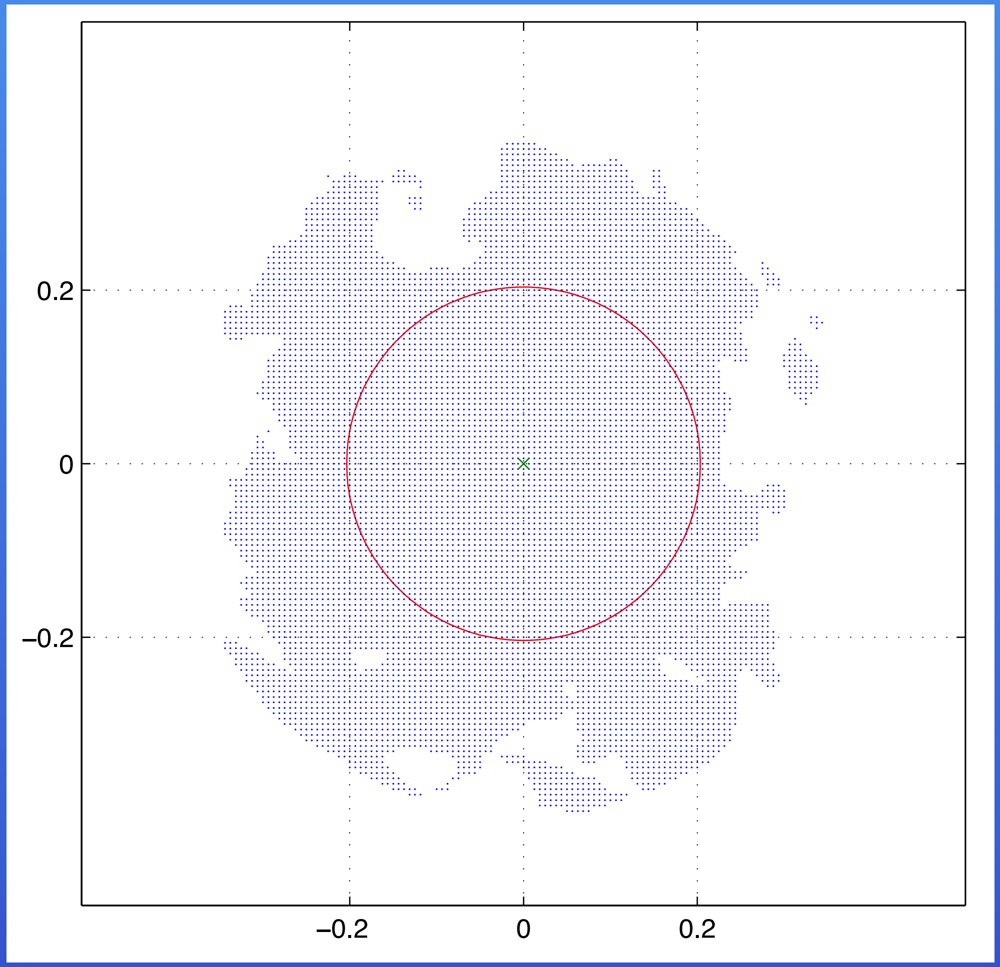


3862 nodes, 38770 links

- Same pattern, different values of the degree centrality
- Structures more extended in space

Circumference (200,600) at z=512

τ=0.5



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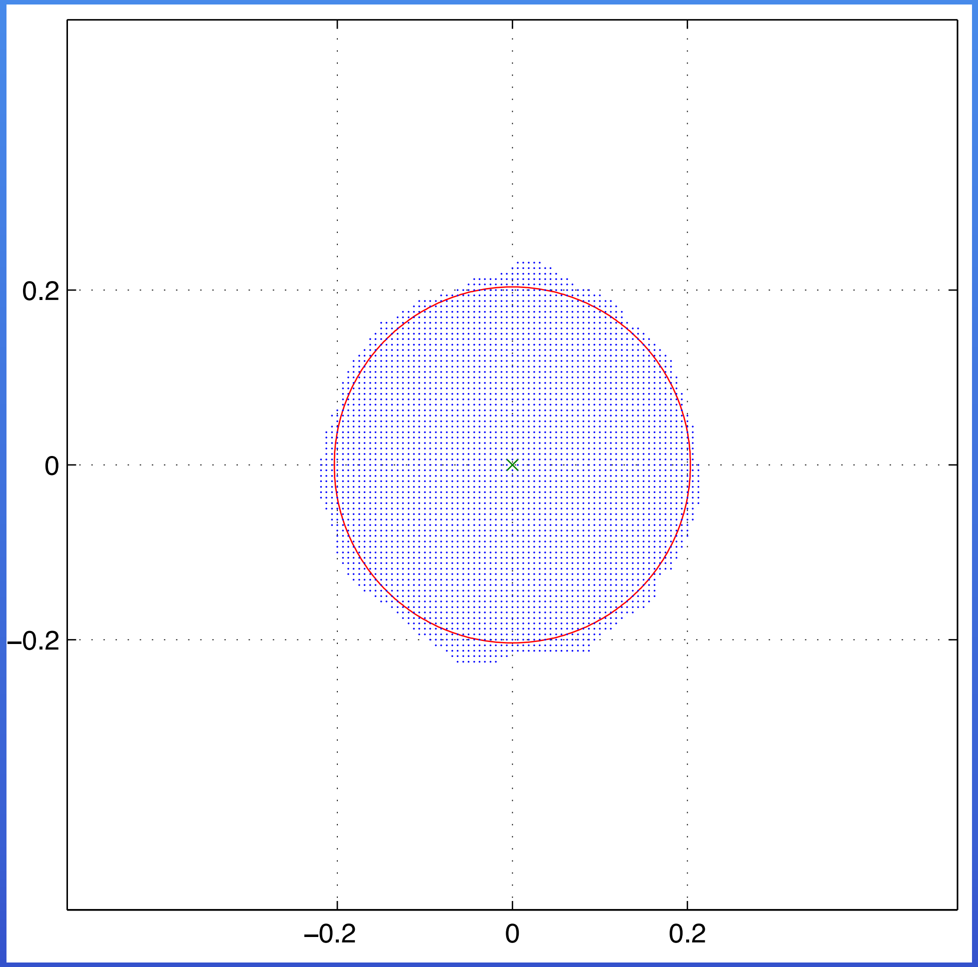
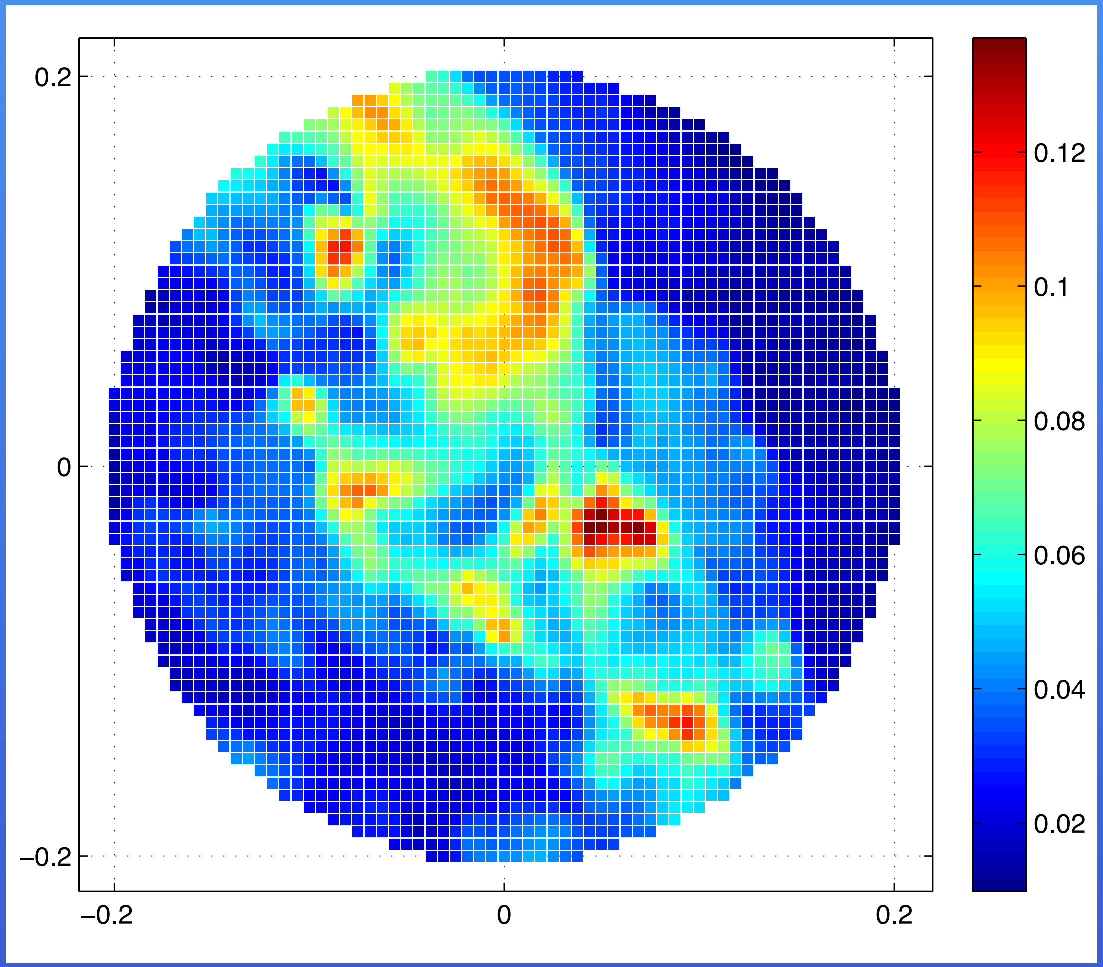
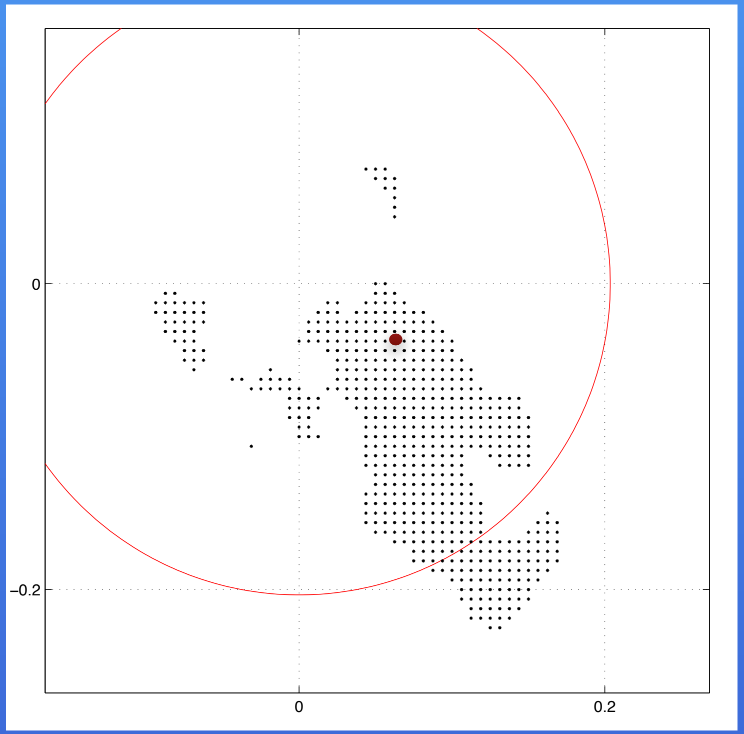


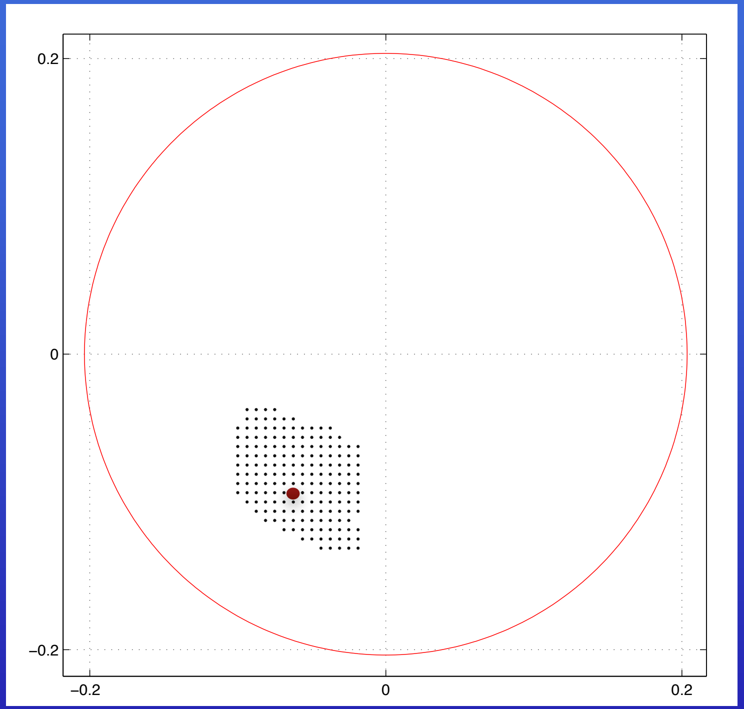
3862 nodes, 38770 links

- Same pattern, different values of the degree centrality
- Structures more extended in space

Circumference (110,350) at z=212, τ=0.9

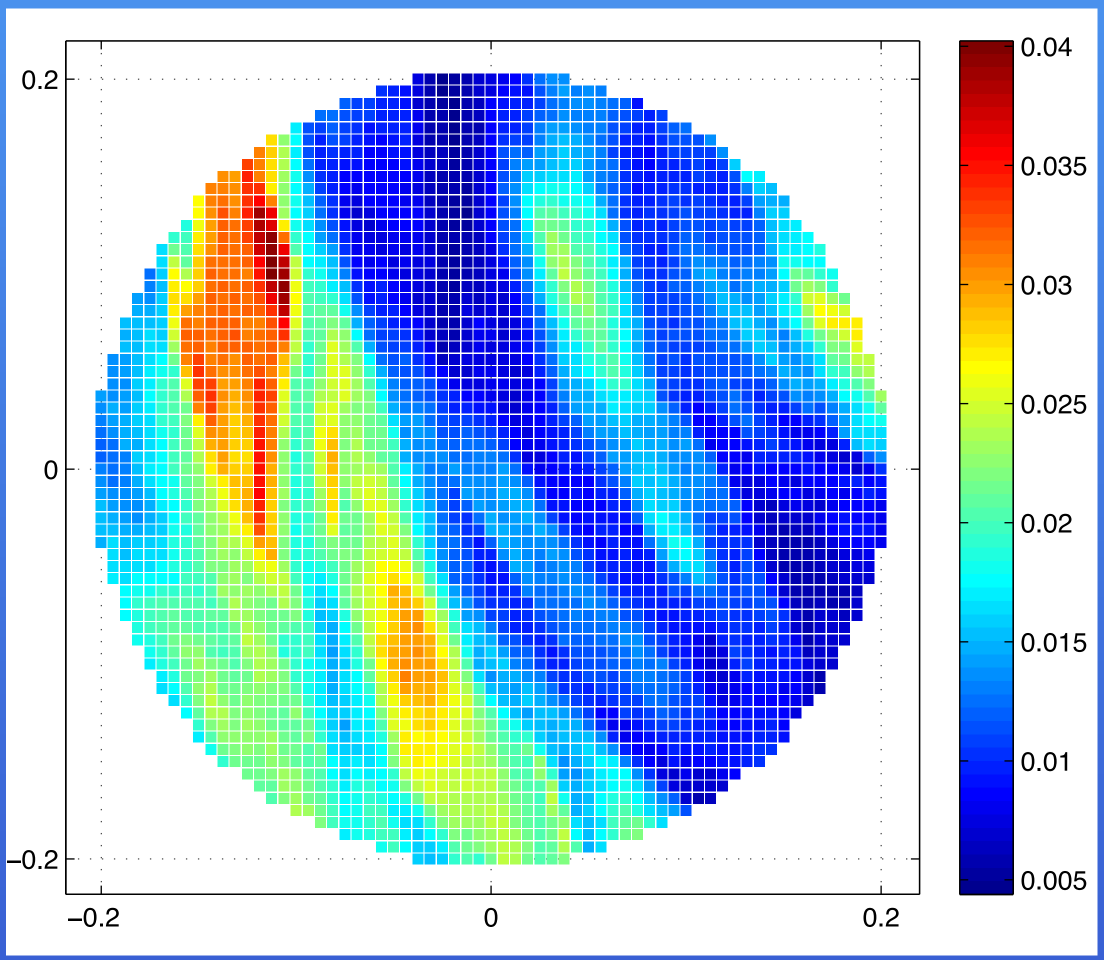




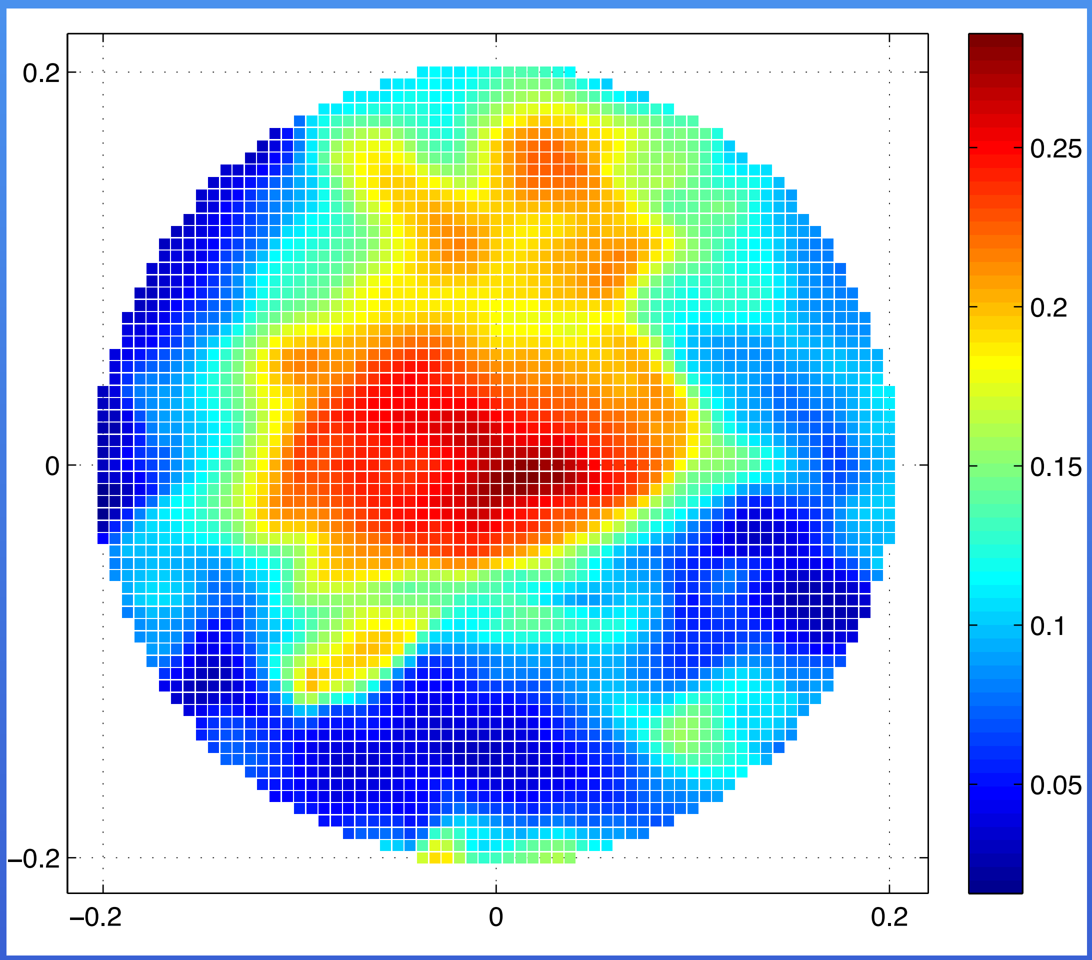


- Nodes 120,344 and 100,335
- Correlation trend different in every direction
- Directional biases lost in 100,335 node

Circumference (390,470) at z=212, τ=0.9



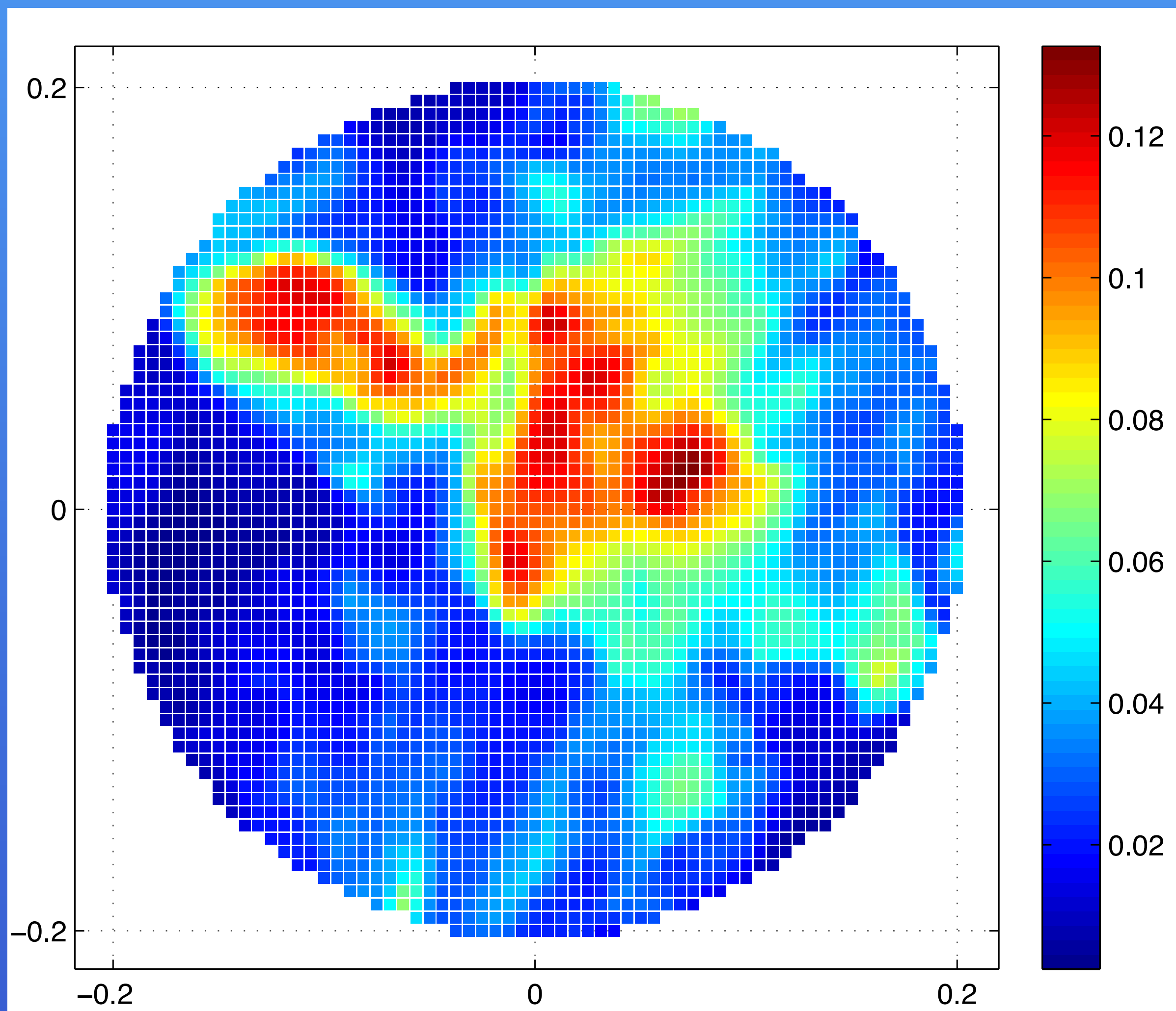
Circumference (510,430) at z=212, τ=0.9



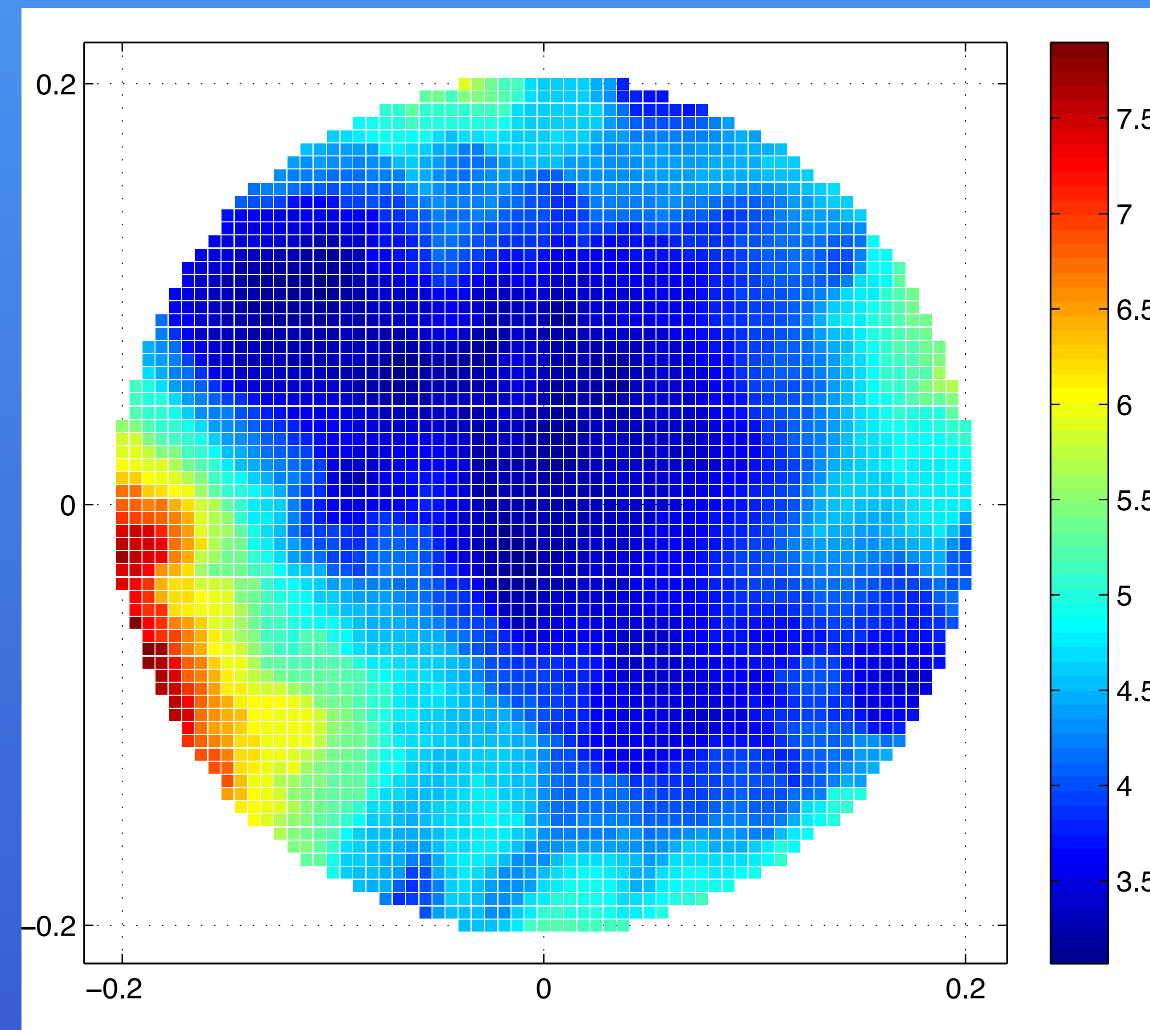
- Spatial homogeneity in less correlated networks
- Streaky pattern typical of networks with a medium value of the degree centrality
- Streaky pattern breaks down in presence of highly correlated nodes

Circumference (280,280) at z=512, τ=0.9

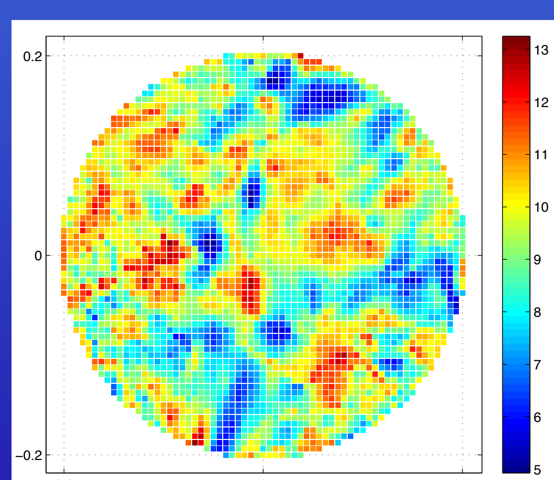
Degree Centrality



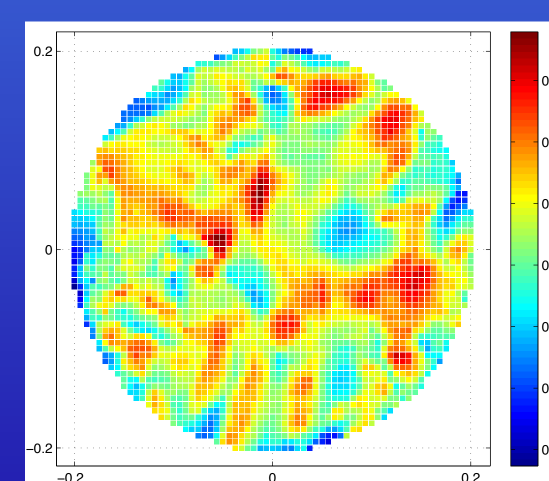
Weighted Average Topological Distance



Betweenness Centrality

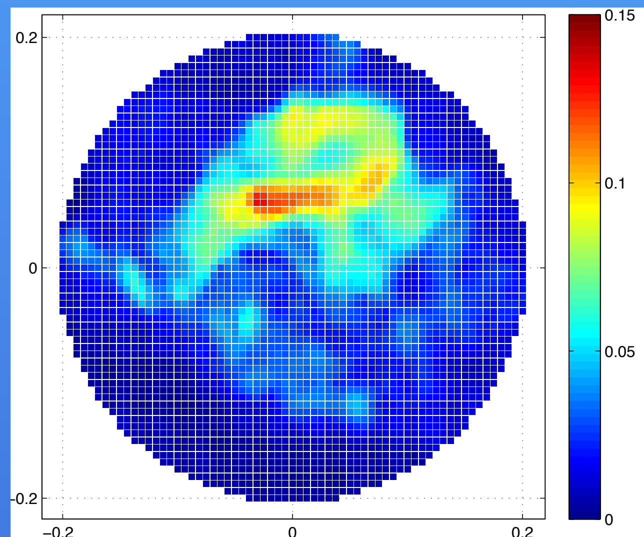


Local Clustering Coefficient

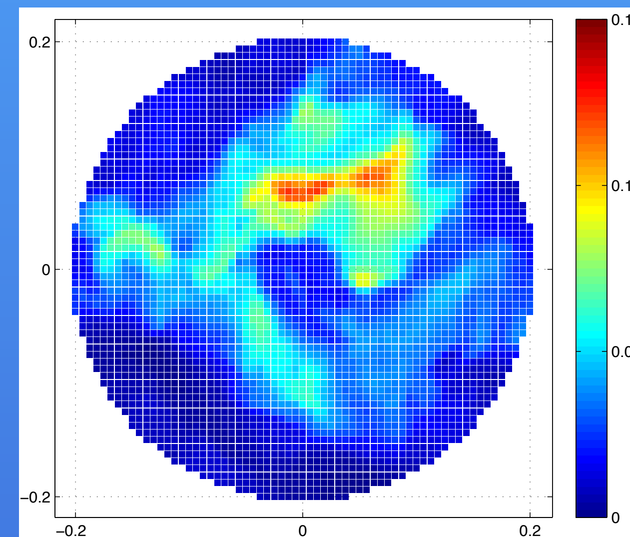


Circumference (280,280), evolution in the z direction

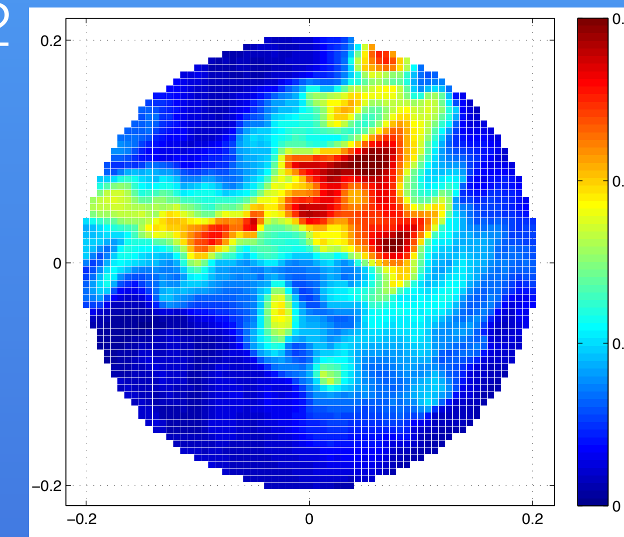
z=492



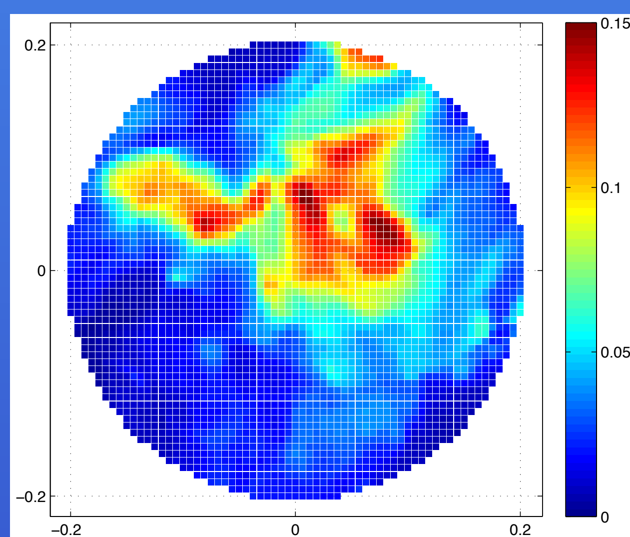
z=497



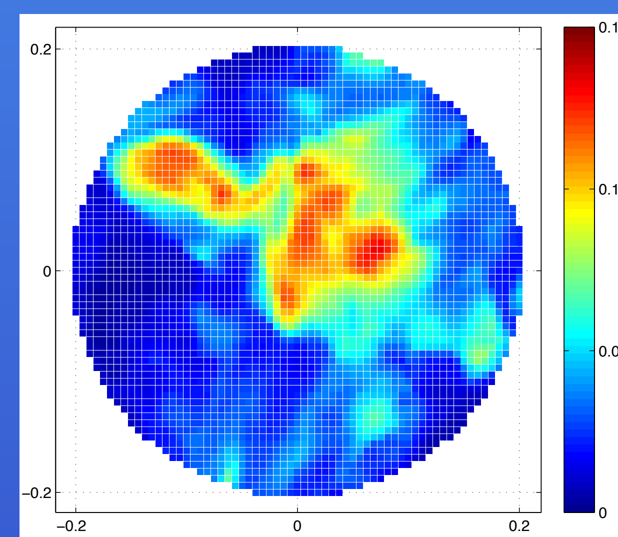
z=502



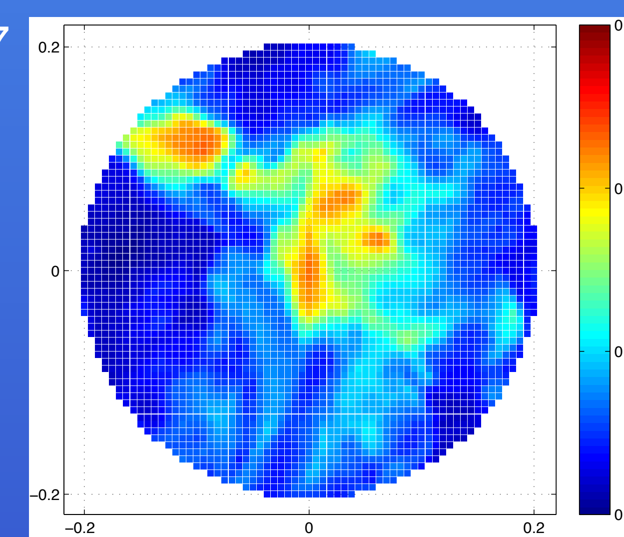
z=507



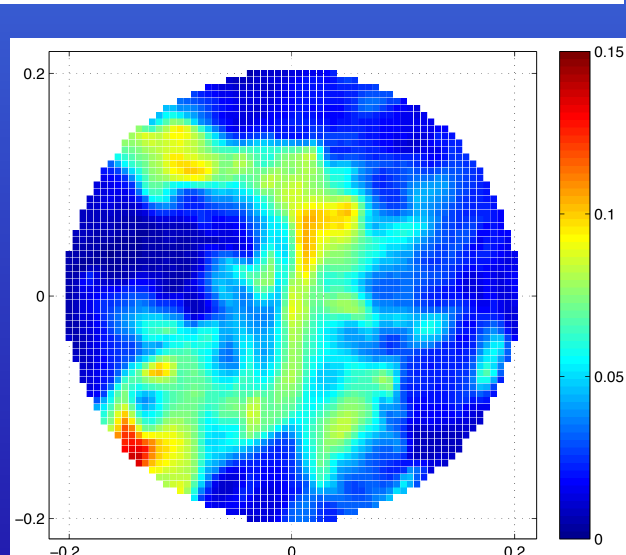
z=512



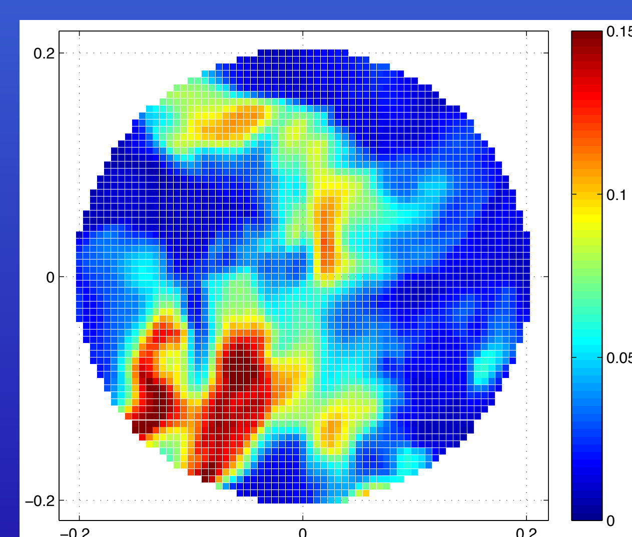
z=517



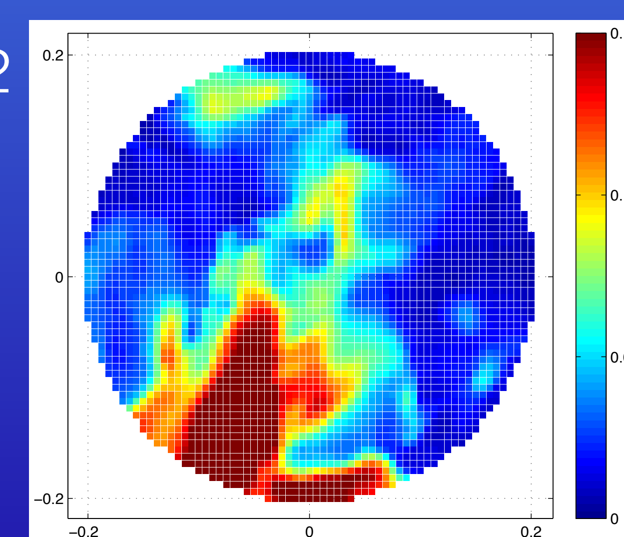
z=522



z=527

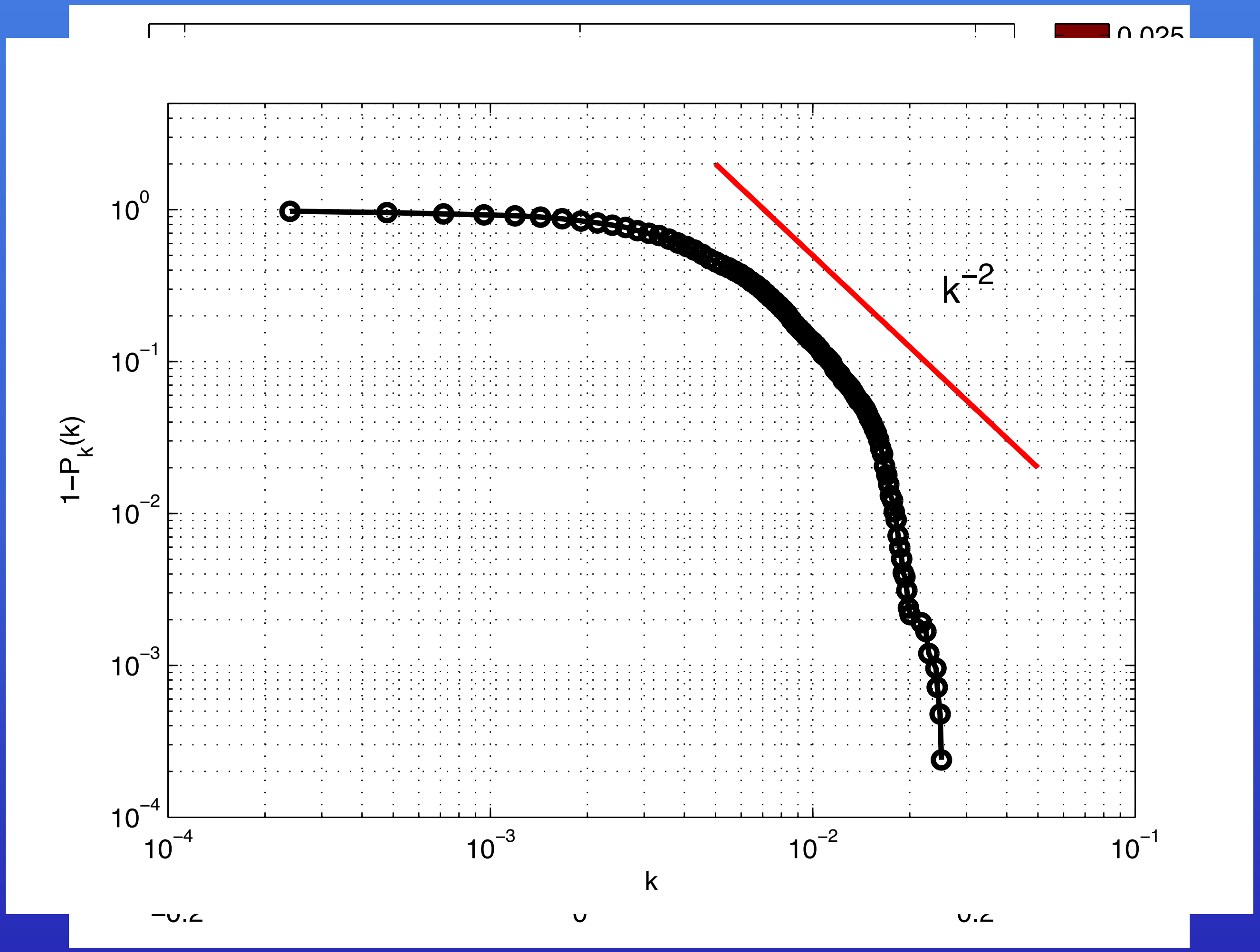


z=532

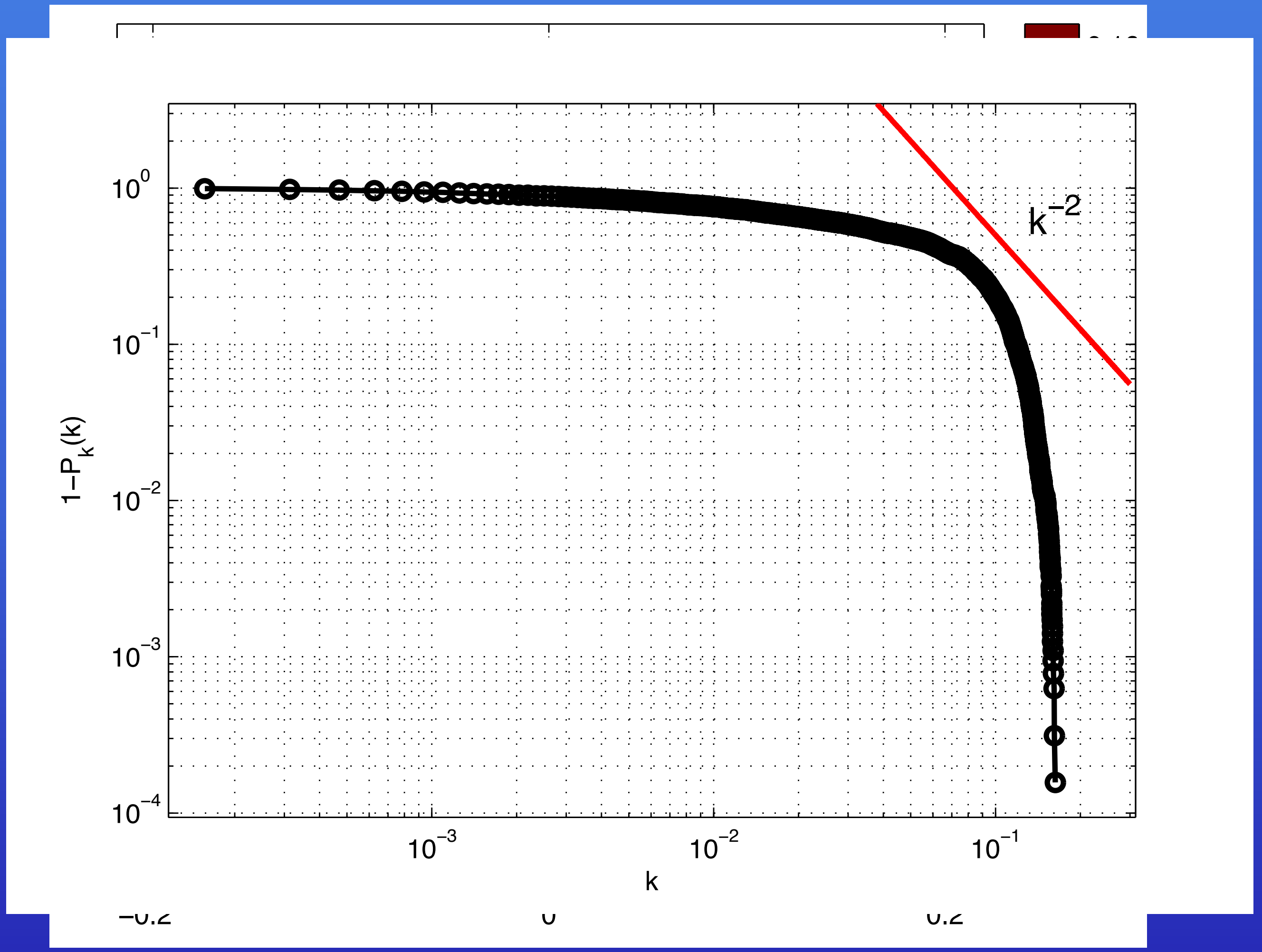


Power-law degree distribution

- Scale-free networks: power law distribution having the same form at all scales
- Real networks are scale-free: few of the nodes are highly correlated while the rest of the network is barely correlated
- Power law ranging from -2 to -3



Circumference (150,230) at z=512



Circumference (730,410) at z=212

Conclusions:

- This turbulent flow shows all the characteristics of a complex network
- The complex network theory may be a complementary approach for the study of turbulence
- Degree centrality and weighted average topological distance may be useful for the spatial characterization of a turbulent flow
- Out of 160 cuts circa, at least 15-20 networks consisting of supernodes (k>0.15) were found
- A highly correlated network may be associated with the presence of energetic patterns. No local isotropy and dishomogeneity

Improvements:

- Turbulence is intrinsically 3D: spherical networks
- Community structures
- Different type of flows, especially the ones with strong dishomogeneity

Thank you for your attention.