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A multiscale approach to study the stability of long waves in near-parallel flows

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Multiple scales analysis

- Different scales in the stability analysis:
  - Slow scales (base flow evolution);
  - Fast scales (disturbance dynamics);

Small parameter is the polar wavenumber of the perturbation:

$k<<1$

- In some flow configurations, long waves can be destabilizing (for example Blasius boundary layer and 3D cross flow boundary layer);

- In such instances the perturbation wavenumber of the unstable wave is much less than O(1).

The initial-value problem formulation

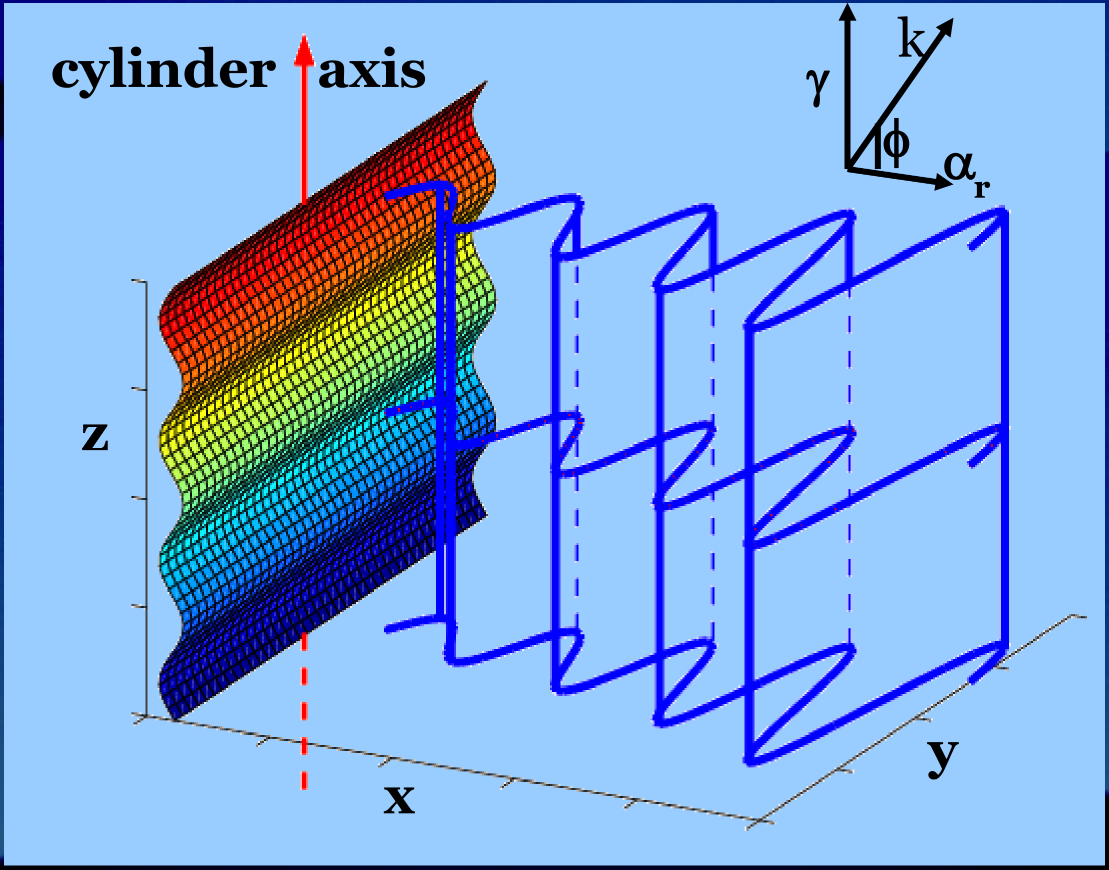
- Linear, three-dimensional perturbative equations in terms of velocity and vorticity (Criminale & Drazin, 1990);
- Laplace-Fourier transform of perturbation quantities in x and z direction (α complex, γ real);

α_r = longitudinal wavenumber

γ = transversal wavenumber

Φ = angle of obliquity

k = polar wavenumber



Full linear system

$$\begin{cases} \dfrac{\partial^2 \hat{v}}{\partial y^2} & - \quad (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{v} = \hat{\Gamma} \\ \dfrac{\partial \hat{\Gamma}}{\partial t} & = \quad G\hat{\Gamma} + H\hat{v} + K\hat{\omega}_y \\ \dfrac{\partial \hat{\omega}_y}{\partial t} & = \quad L\hat{\omega}_y + M\hat{v} \end{cases}$$

$$\tilde{\omega}_y = \frac{\partial \tilde{u}}{\partial z} - \frac{\partial \tilde{w}}{\partial x}$$

$$\tilde{\Gamma} = \frac{\partial \tilde{\omega}_z}{\partial x} - \frac{\partial \tilde{\omega}_x}{\partial z}$$

$$G = G(y, t; k, \phi, \alpha_i, R, U, V), \quad (U(x_0, y; R), V(x_0, y; R))$$

- Initial conditions periodic and bounded in the free stream

$$\hat{\omega}_y(y, t = 0) = 0 \begin{cases} \hat{\Gamma}(y, t = 0) = e^{-(y-y_0)^2} sin(\beta_0(y - y_0)) & \text{asymmetric} \\ \qquad \text{or} & \\ \hat{\Gamma}(y, t = 0) = e^{-(y-y_0)^2} cos(\beta_0(y - y_0)) & \text{symmetric} \end{cases}$$

Multiple scales hypothesis

- Regular perturbation scheme, k<<1:

$$\hat{v} = \hat{v}_0 + k\hat{v}_1 + k^2\hat{v}_2 + \ldots$$
$$\hat{\Gamma} = \hat{\Gamma}_0 + k\hat{\Gamma}_1 + k^2\hat{\Gamma}_2 + \ldots$$
$$\hat{\omega}_y = \hat{\omega}_{y0} + k\hat{\omega}_{y1} + k^2\hat{\omega}_{y2} + \ldots$$

- Temporal scales: $t, \tau = kt, T = k^2 t;$
- Spatial scales: $y, Y = ky;$

Order O(1)

$$\begin{cases} \dfrac{\partial^2 \hat{v}_0}{\partial y^2} + \alpha_i^2 \hat{v}_0 = \hat{\Gamma}_0 \\ \dfrac{\partial \hat{\Gamma}_0}{\partial t} = G_f \hat{\Gamma}_0 + H_f \hat{v}_0 \\ \dfrac{\partial \hat{\omega}_{y0}}{\partial t} = L_f \hat{\omega}_{y0} \end{cases}$$

$$G_f = G_f(y, t; \phi, \alpha_i, R, U, V)$$

Order O(k)

$$\begin{cases} \frac{\partial^2 \hat{v}_1}{\partial y^2} + \alpha_i^2 \hat{v}_1 &= -2\frac{\partial^2 \hat{v}_0}{\partial y \partial Y} + 2icos(\phi)\alpha_i \hat{v}_0 + \hat{\Gamma}_1 \\ \frac{\partial \hat{\Gamma}_1}{\partial t} + \frac{\partial \hat{\Gamma}_0}{\partial \tau} &= G_f \hat{\Gamma}_1 + H_f \hat{v}_1 + G_{sf}\hat{\Gamma}_0 + H_{sf}\hat{v}_0 + K_{sf}\hat{\omega}_{y0} \\ \frac{\partial \hat{\omega}_{y1}}{\partial t} + \frac{\partial \hat{\omega}_{y0}}{\partial \tau} &= L_f \hat{\omega}_{y1} + L_{sf}\hat{\omega}_{y0} + M_{sf}\hat{v}_0 \end{cases}$$

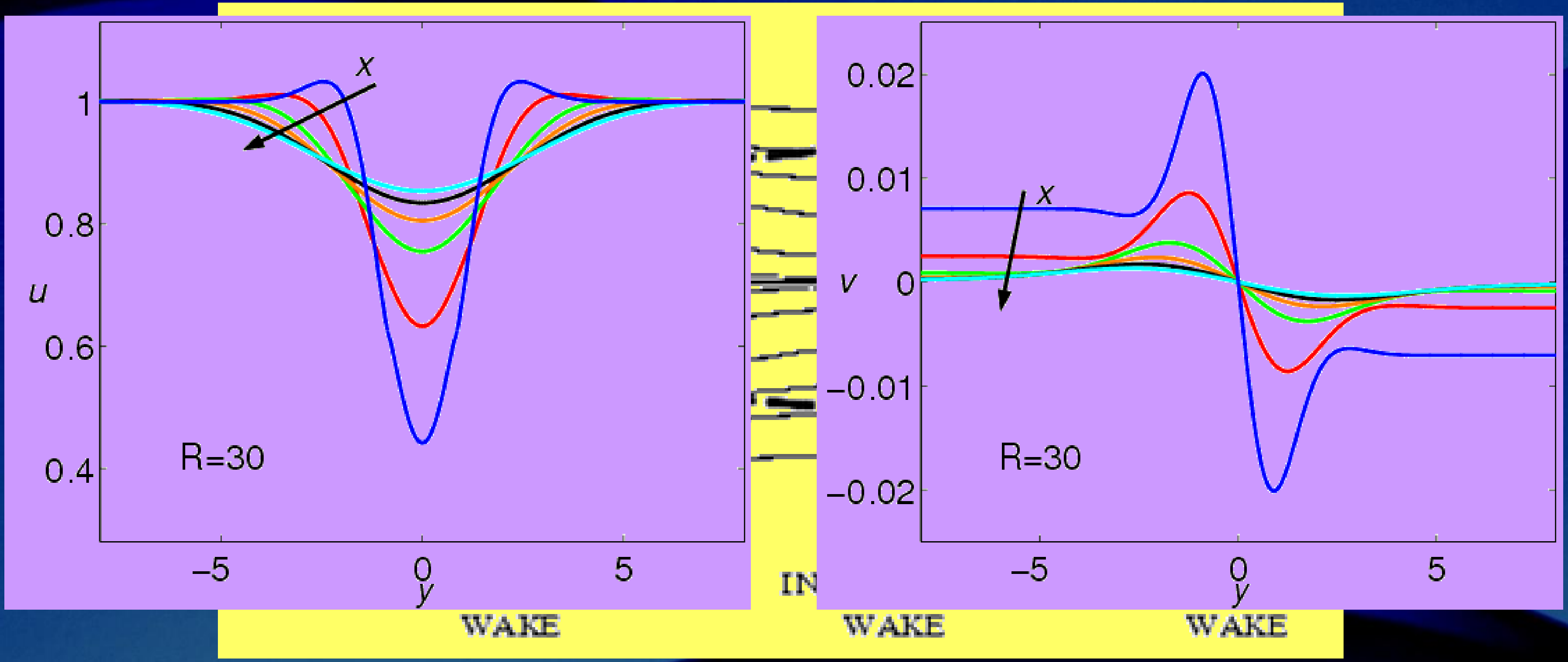
$$G_{sf} = G_{sf}(y, t, Y, \tau; \phi, \alpha_i, R, U, V)$$

- Initial conditions at order O(1) defined as in the full linear problem and at order O(k), O(k²),... equal to zero:

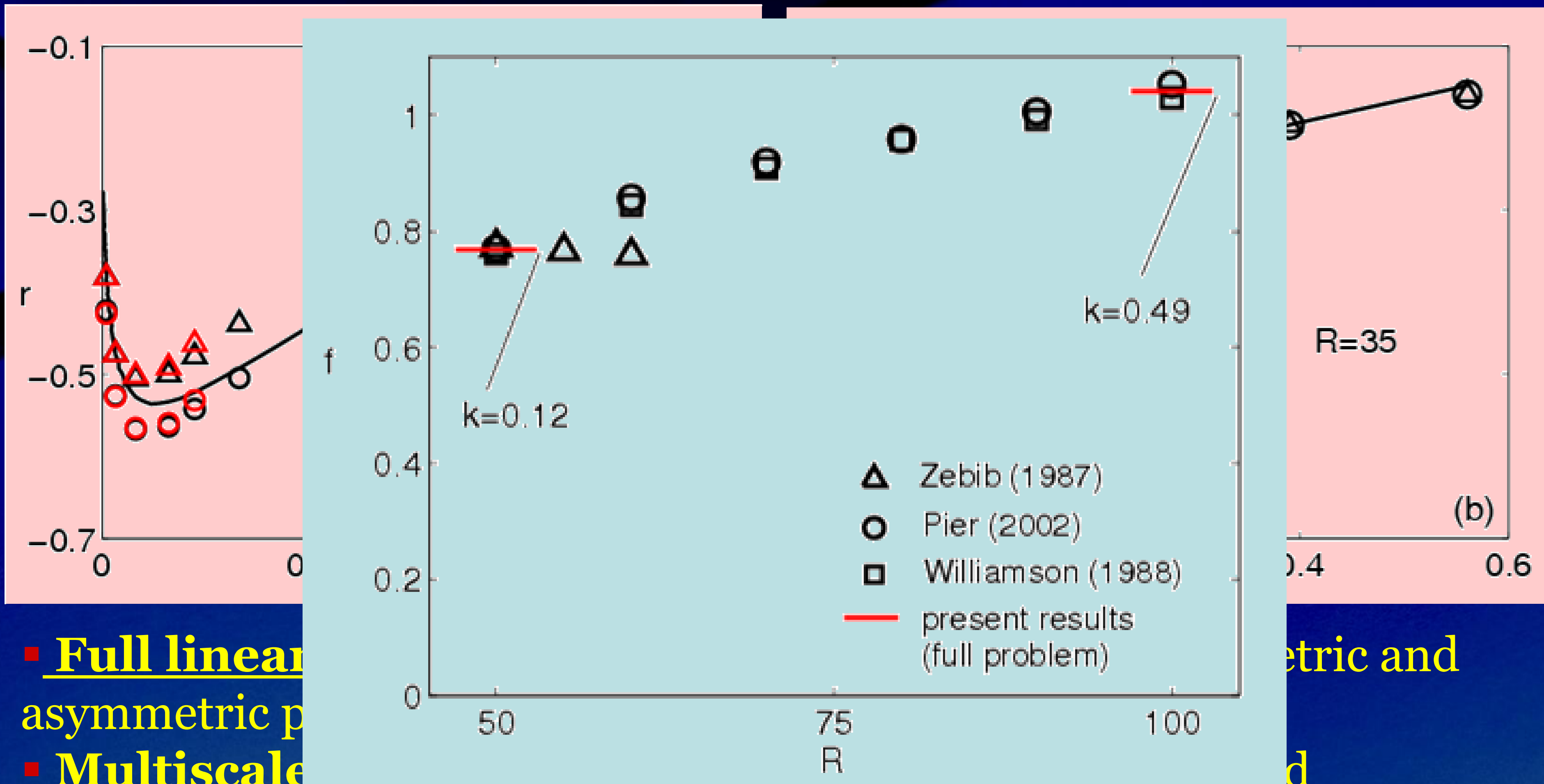
$$\begin{aligned} \hat{\Gamma}_0(y, Y, 0, 0, 0; k, \phi) &= \hat{\Gamma}(y, 0; k, \phi), \\ \hat{\Gamma}_1(y, Y, 0, 0, 0; k, \phi) &= \hat{\Gamma}_2(y, Y, 0, 0, 0; k, \phi) = \cdots = 0, \\ \hat{\omega}_{y0}(y, Y, 0, 0, 0; k, \phi) &= \hat{\omega}_y(y, 0; k, \phi), \\ \hat{\omega}_{y1}(y, Y, 0, 0, 0; k, \phi) &= \hat{\omega}_{y2}(y, Y, 0, 0, 0; k, \phi) = \cdots = 0. \end{aligned}$$

Validation of the multiscale analysis

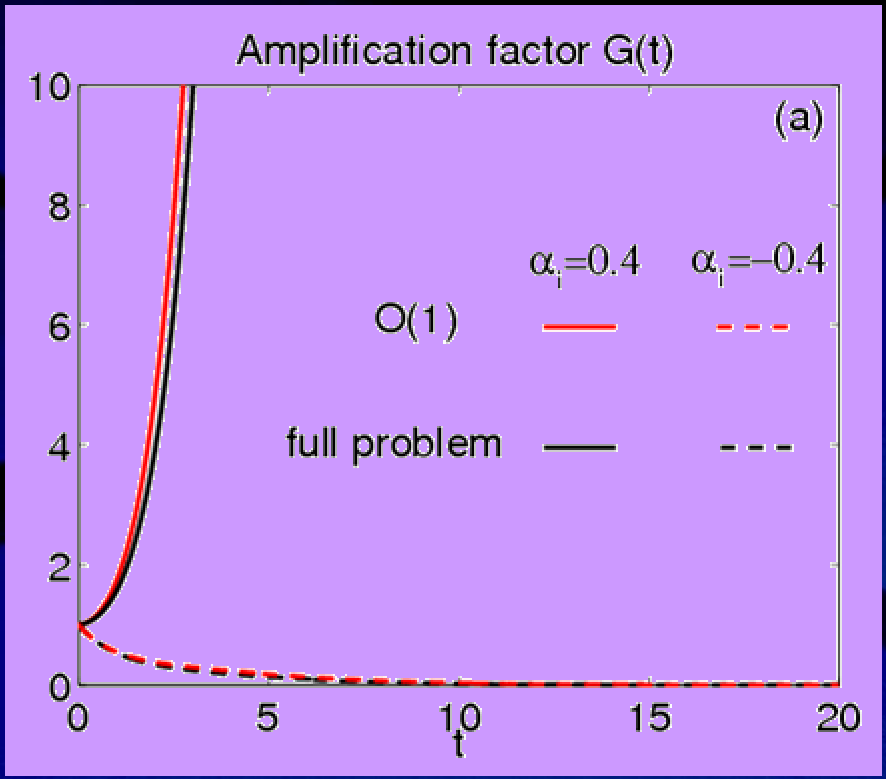
- Flow behind a circular cylinder → steady, incompressible and viscous;
- Approximation of 2D asymptotic Navier-Stokes expansions (Belan & Tordella, 2003) → weakly non-parallel flow (U(x,y;R), V(x,y;R))



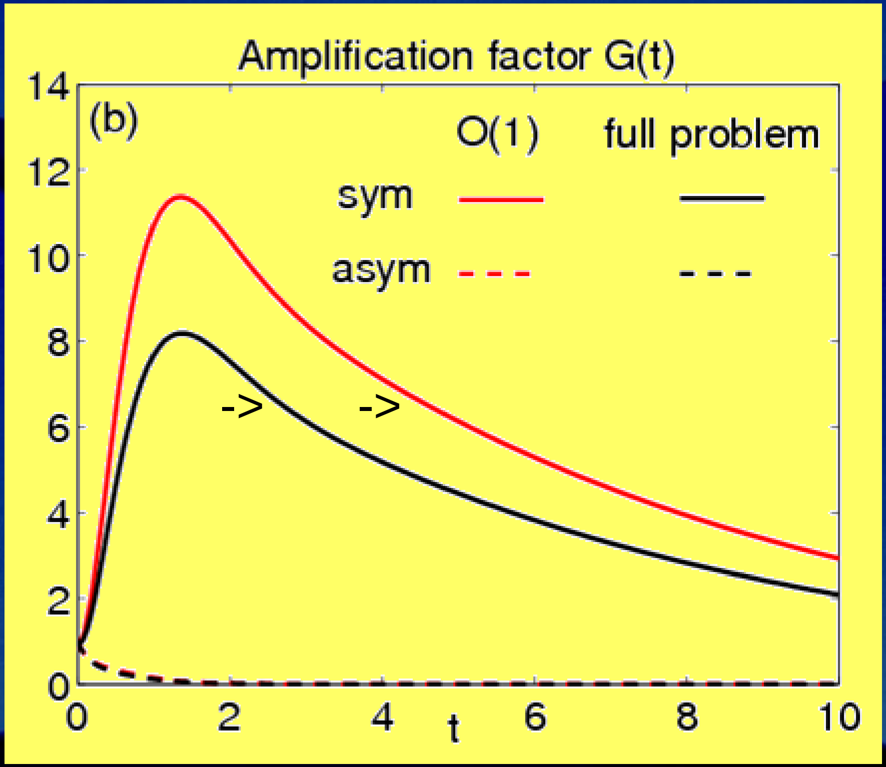
Results



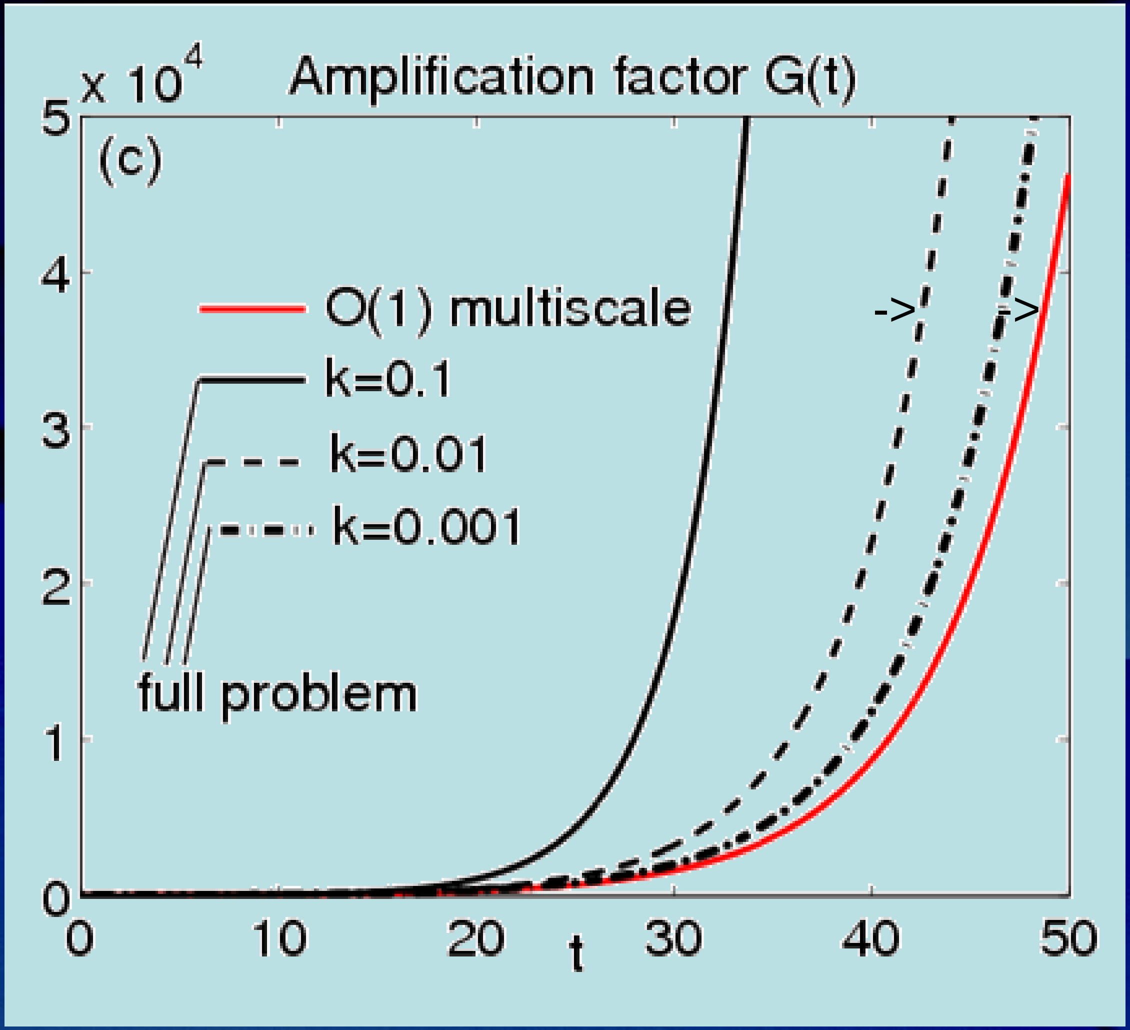
- **Full linear** ... etric and asymmetric p...
- **Multiscale** ... d asymmetric perturbations).
- **Normal mode analysis**: solid lines (Tordella, Scarsoglio and Belan, *Phys. Fluids* 2006).



(a): R=50, y_o=0, k=0.03, β_o=1, x_o=12, asymmetric initial condition, $\Phi= \pi/4$, α_i=-0.4, 0.4.



(b): R=100, y_o=0, k=0.06, β_o=1, x_o=14.50, $\Phi= (3/8)\pi$, α_i=-1.9, symmetric and asymmetric perturbations.



(c): R=100, y_o=0, β_o=1, x_o=9, symmetric initial condition, Φ=0, α_i=-1.7, k=0.1, 0.01, 0.001.

Conclusions

- Multiple scales approach for long waves can be applied to the stability analysis of shear flows in general;
- Different transient configurations in agreement with the full linear problem;
- Asymptotically good agreement with full linear problem and normal mode theory;
- Possible extension to O(k) order.