Energy spectrum in shear flows. A general pre-unstable large set of multiple transient three-dimensional waves and the turbulent state

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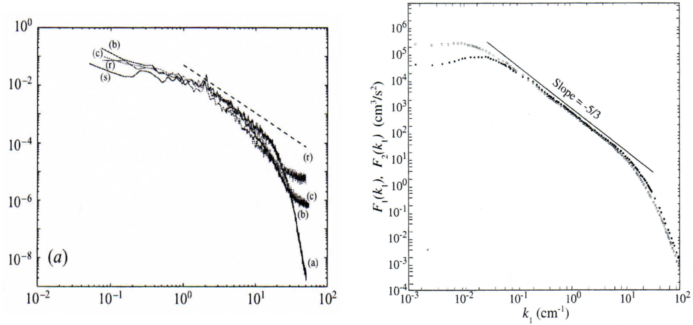

(left) *Evangelinos & Karniadakis, JFM 1999*. (right) *Champagne, JFM 1978*.

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  - Leaves aside the nonlinear interaction among the different scales;
- The perturbative evolution is ruled out by the **initial-value problem** associated to the Navier-Stokes linearized formulation.

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  - The difference is large $\Rightarrow$ quantitative measure of the nonlinear interaction in spectral terms;
  - The difference is small $\Rightarrow$ higher degree of universality on the value of the exponent of the inertial range, not necessarily associated to the nonlinear interaction.

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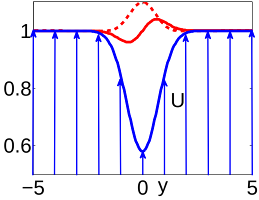

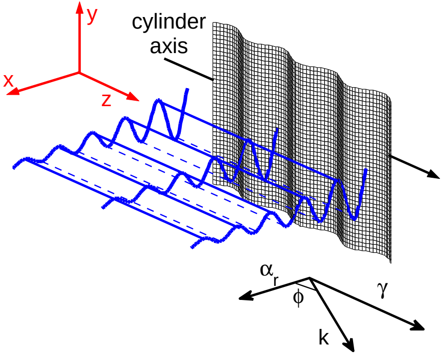

Perturbative equations

- Perturbative linearized system:

$$
\begin{aligned}
\frac{\partial^2 \hat{v}}{\partial y^2} &- (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{v} = \hat{\Gamma} \\
\frac{\partial \hat{\Gamma}}{\partial t} &= (i\alpha_r - \alpha_i)(\frac{d^2U}{dy^2}\hat{v} - U\hat{\Gamma}) + \frac{1}{Re}[\frac{\partial^2 \hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\Gamma}] \\
\frac{\partial \hat{\omega}_y}{\partial t} &= -(i\alpha_r - \alpha_i)U\hat{\omega}_y - i\gamma\frac{dU}{dy}\hat{v} + \frac{1}{Re}[\frac{\partial^2 \hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\omega}_y]
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  The transversal velocity and vorticity components are $\hat{v}$ and $\hat{\omega}_y$ respectively, $\hat{\Gamma}$ is defined as $\widetilde{\Gamma} = \partial_x\widetilde{\omega}_z - \partial_z\widetilde{\omega}_x$.

- Initial conditions:
  - $\hat{\omega}_y(0, y) = 0$;
  - $\hat{v}(0, y) = e^{-y^2}\sin(y)$ or $\hat{v}(0, y) = e^{-y^2}\cos(y)$;
- Boundary conditions: $(\hat{u}, \hat{v}, \hat{w}) \to 0$ as $y \to \infty$.

Perturbation energy

- Kinetic energy density e:

$$
\begin{aligned}
e(t;\alpha,\gamma) &= \int_{-y_d}^{+y_d} (|\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2)dy \\
&= \frac{1}{|\alpha^2+\gamma^2|}\int_{-y_d}^{+y_d} (|\frac{\partial \hat{v}}{\partial y}|^2 + |\alpha^2+\gamma^2||\hat{v}|^2 + |\hat{\omega}_y|^2)dy
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- Temporal growth rate r (*Lasseigne et al., J. Fluid Mech., 1999*):

$$r(t;\alpha,\gamma) = \frac{log|e(t;\alpha,\gamma)|}{2t}, \quad t>0$$

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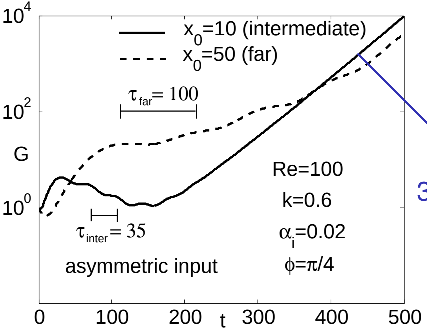
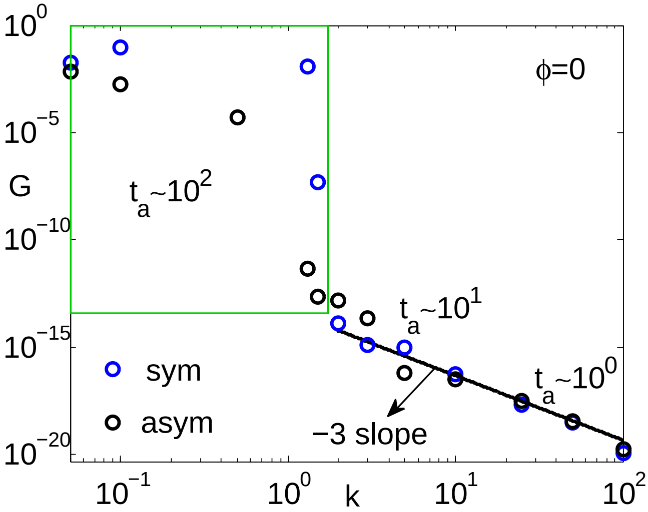
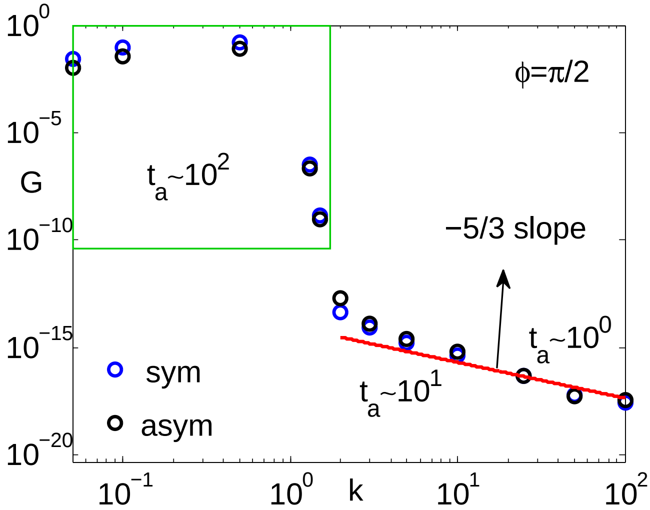


3D Visualization

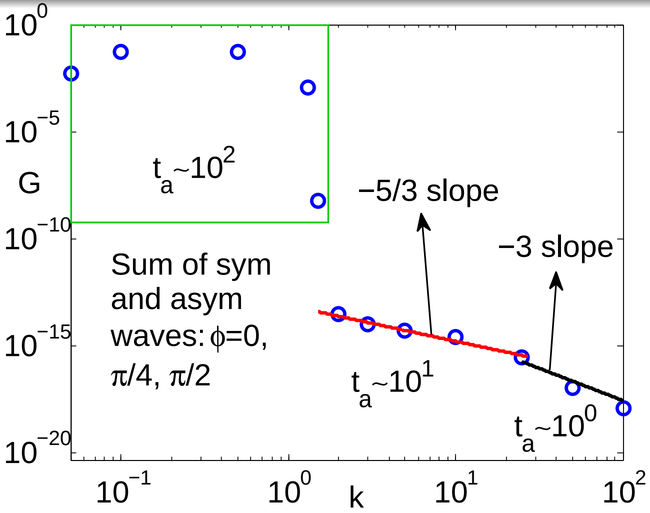
Energy spectrum for longitudinal waves



Energy spectrum for transversal waves



Energy spectrum of a 2D-3D combined perturbation



Transient evolution of multiple three-dimensional waves.

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Coming next $\Rightarrow$ Temporal observation window of a large number of small 3D perturbations injected in a statistical way into the system.