Politecnico di Torino - Turin (Italy)
Aerospace Engineering Department

Characterization of turbulent channel flows:

From time-series to complex networks

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Co-Authors: Stefania Scarsoglio, Luca Ridolfi

Introduction

Data: Numerical simulations / experimental measures

➡ Fully developed turbulent channel flow

Method: Application of complex networks to time-series

➡ Visibility Algorithm

Aim: Novel method to analyze turbulence data

➡ Extract non-trivial information: temporal structure

Complex Networks

Statistical Physics + Graph Theory → Nodes + Links

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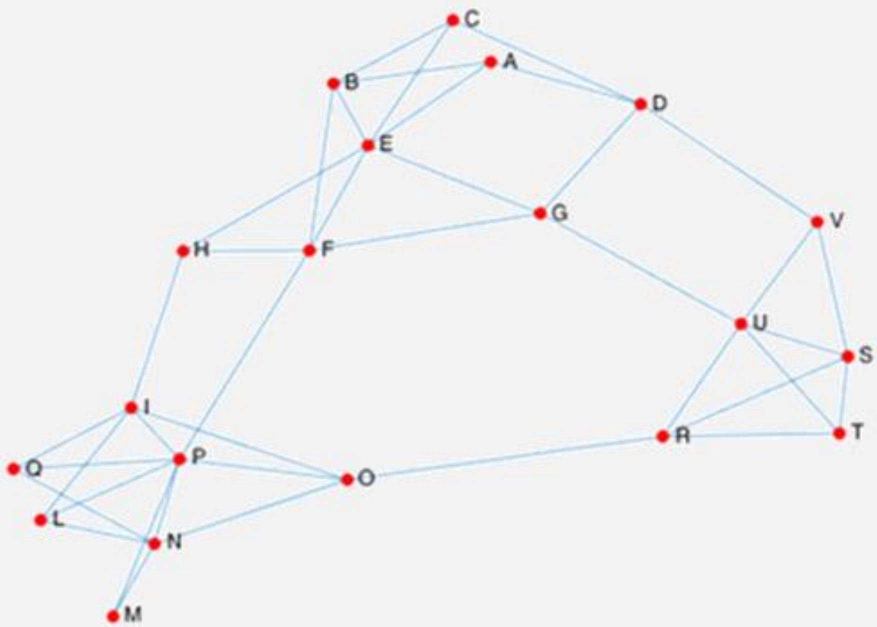

Non-trivial topological features

Complex Networks

Statistical Physics + Graph Theory ➡ Nodes + Links

Successfully applied:

- Transportation
- Internet / WWW
- Biology/Medicine
- Economy
- Earth science
- Sociology
- Fluid Flows

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- Fluid Flows ➡ Turbulence **Very recently**

Networks Building

Each series value ≡ a node

How are nodes linked together?

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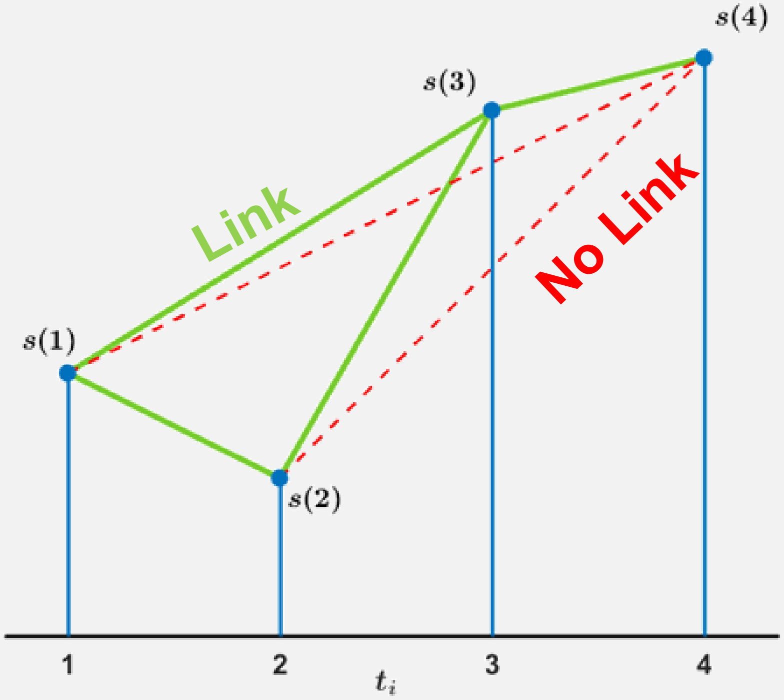
Visibility Algorithm [2]

$$s(t_k) < s(t_j) + (s(t_i) - s(t_j))\,\frac{t_j - t_k}{t_j - t_i}$$



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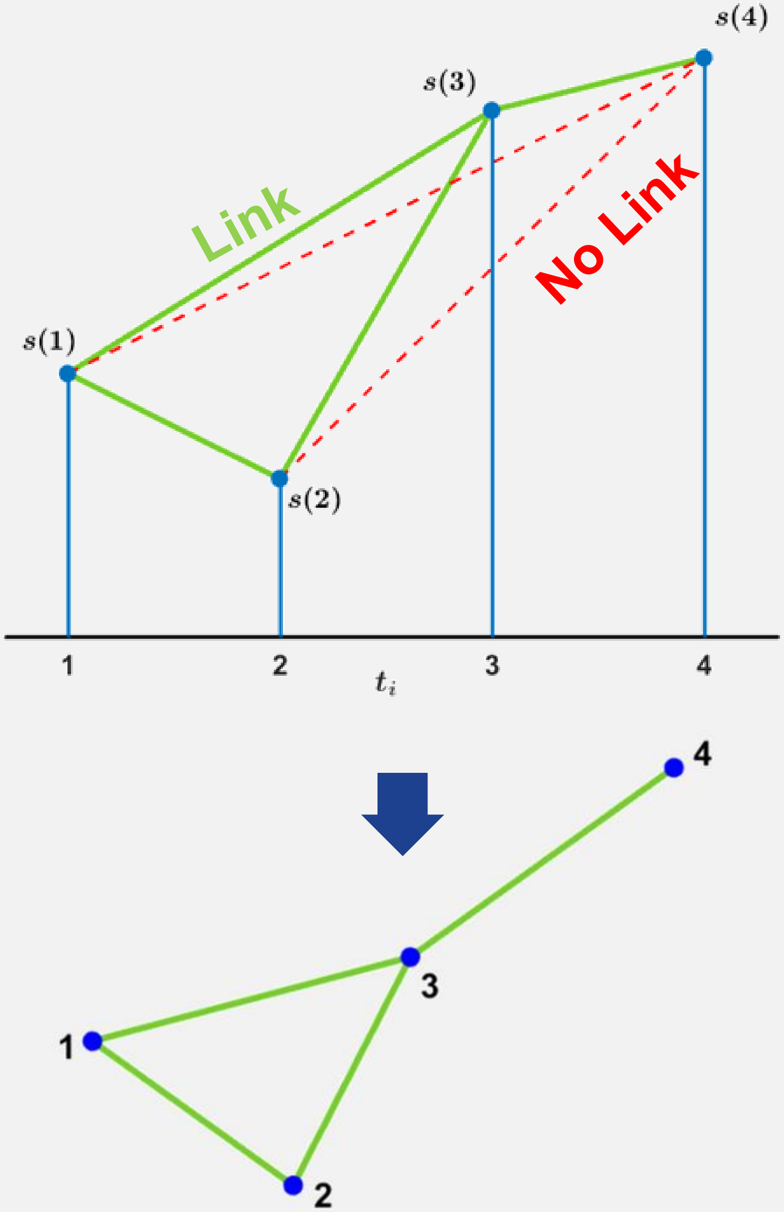

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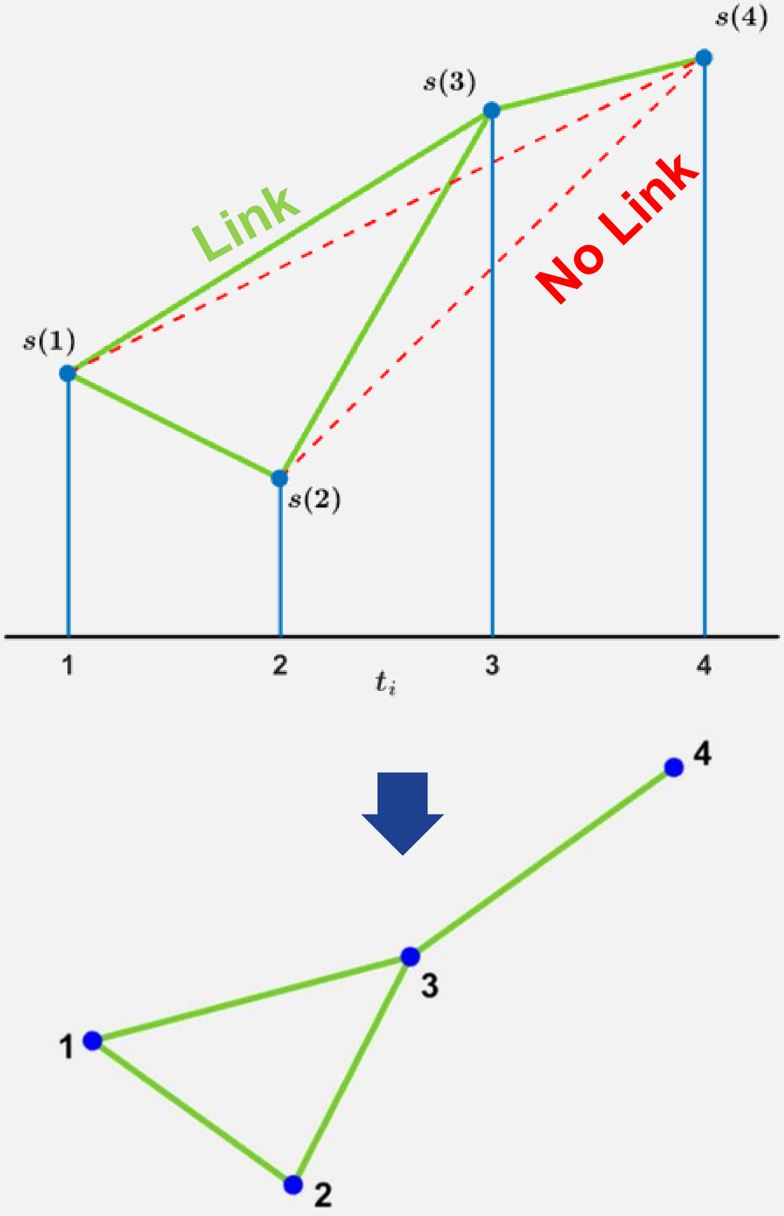
Invariant under affine transformations!



Network Metrics

Global metrics: one value for each network

- Average Degree Centrality, k
- Transitivity, Tr
- Mean Link-Length, d_{1n}

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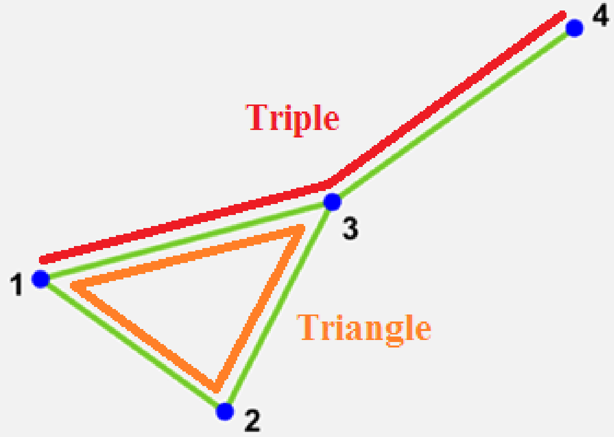

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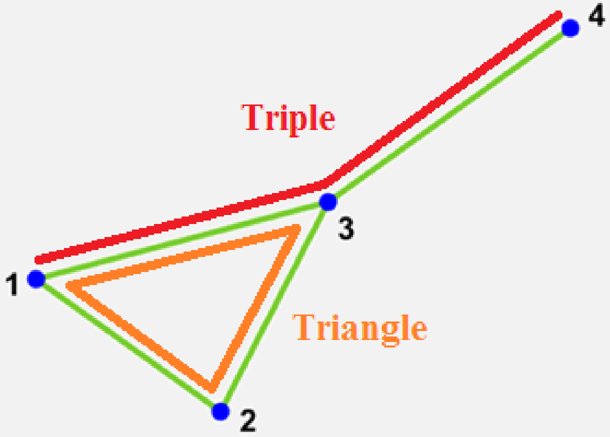
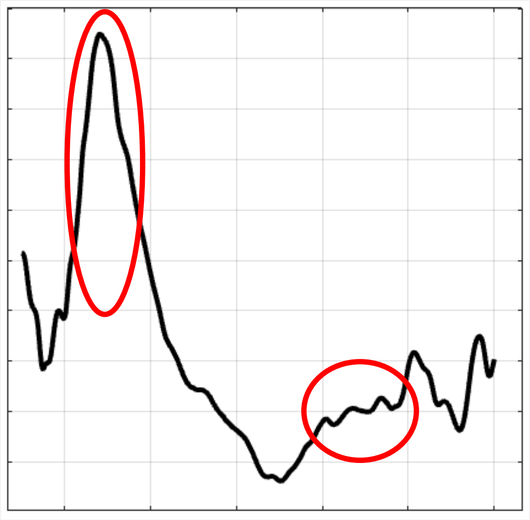
- Mean Link-Length, d_{1n}
  - Average temporal distance among nodes



Network Analysis

Focus on:

- *Peaks* (strong variations)
- *Irregularities* (small variations)



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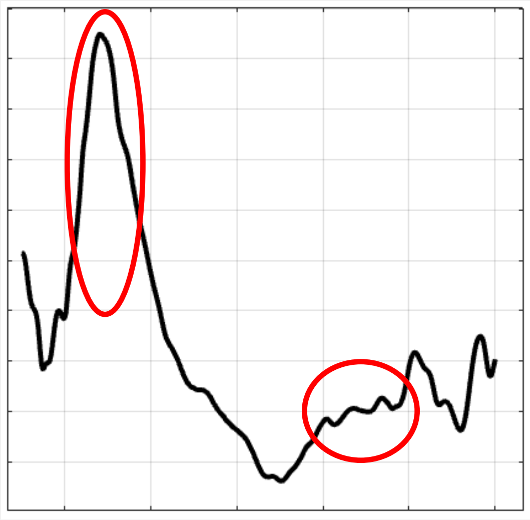
d_{1n} and Tr } k



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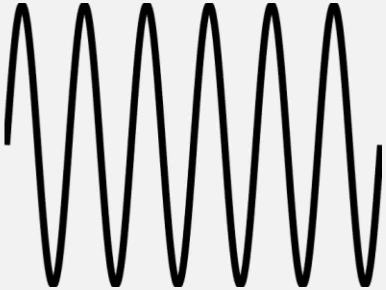
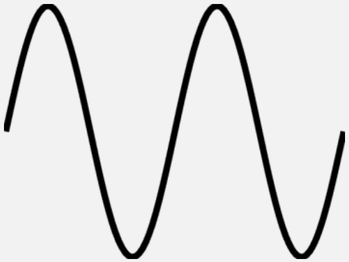
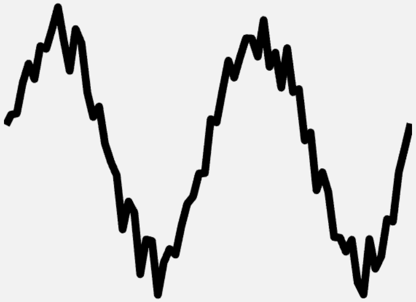
Comparing the temporal structure of pairs of time-series







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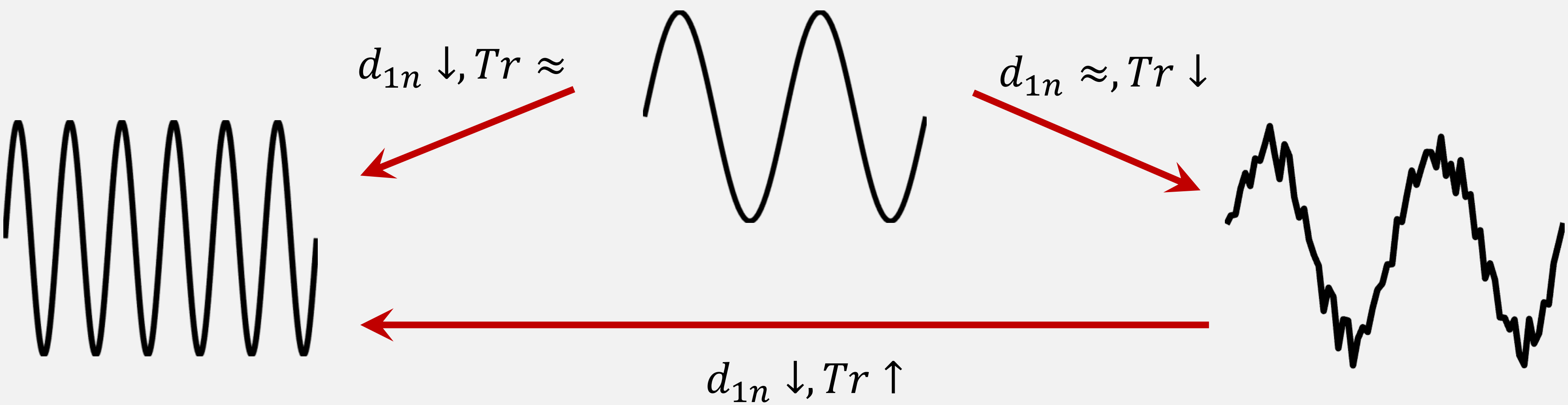

Data & Processing

John Hopkins Turbulence Database [1]

- DNS: $Re_\tau = 1000$
- Grid Resolution: $(2048 \times 512 \times 1536) \rightarrow (64 \times 70 \times 12)$
- Time samples: $N_T = 4000 \rightarrow$ 4000 nodes

[1] http://turbulence.pha.jhu.edu/

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- A network for each grid point $\rightarrow$ 3 values of metrics
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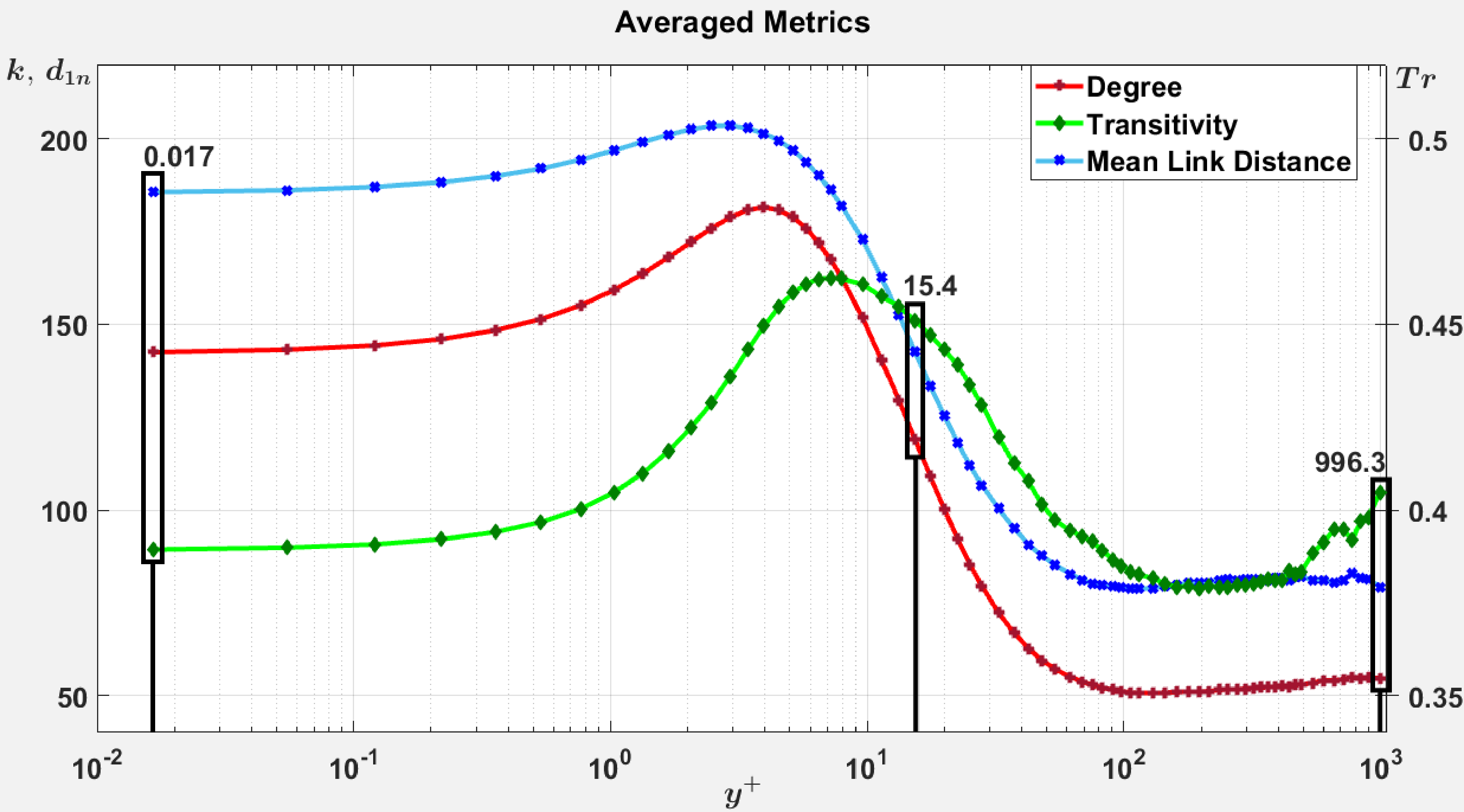
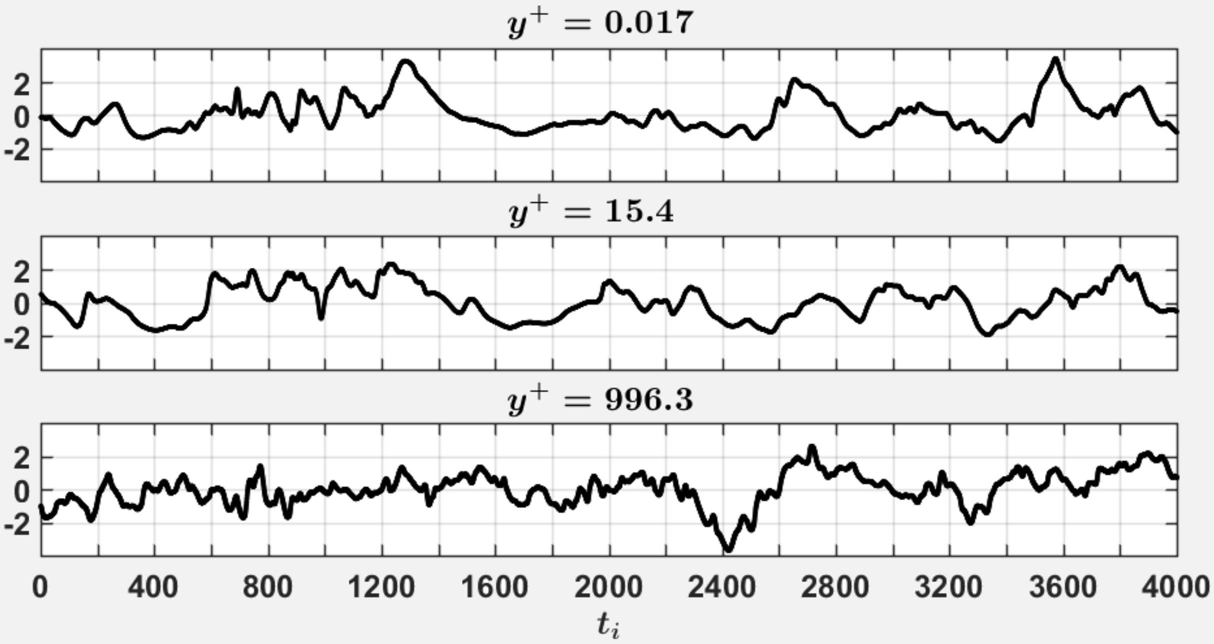
Results: u^*



Averaged Metrics
k, d1n
Tr
Degree
Transitivity
Mean Link Distance
0.017
15.4
996.3
200
150
100
50
0.5
0.45
0.4
0.35
10-2
10-1
100
101
102
103
y+

Results: u^*

Normalized time-series – $u^*(t_i)$



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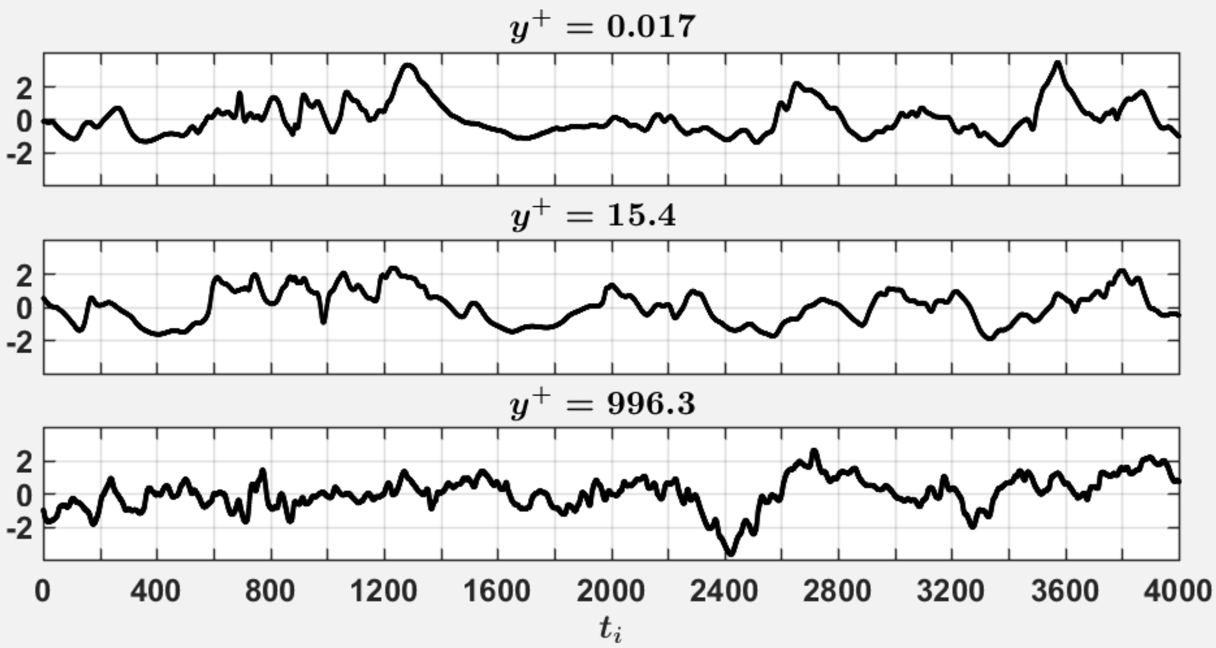

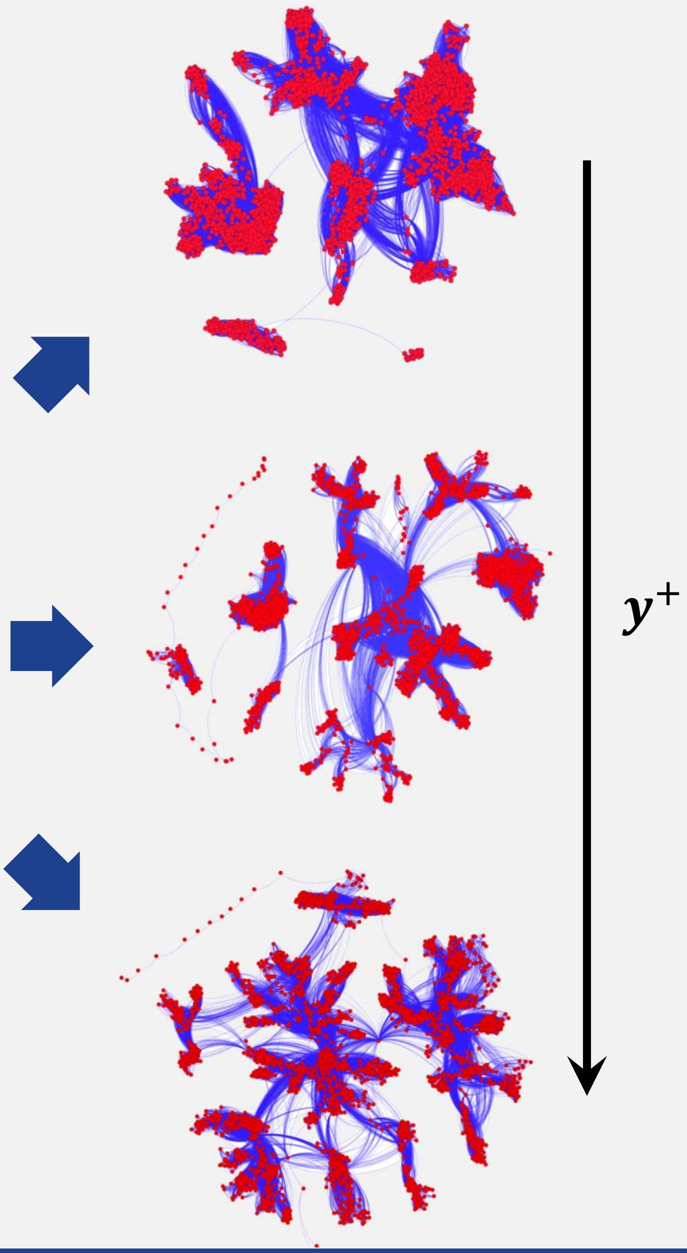

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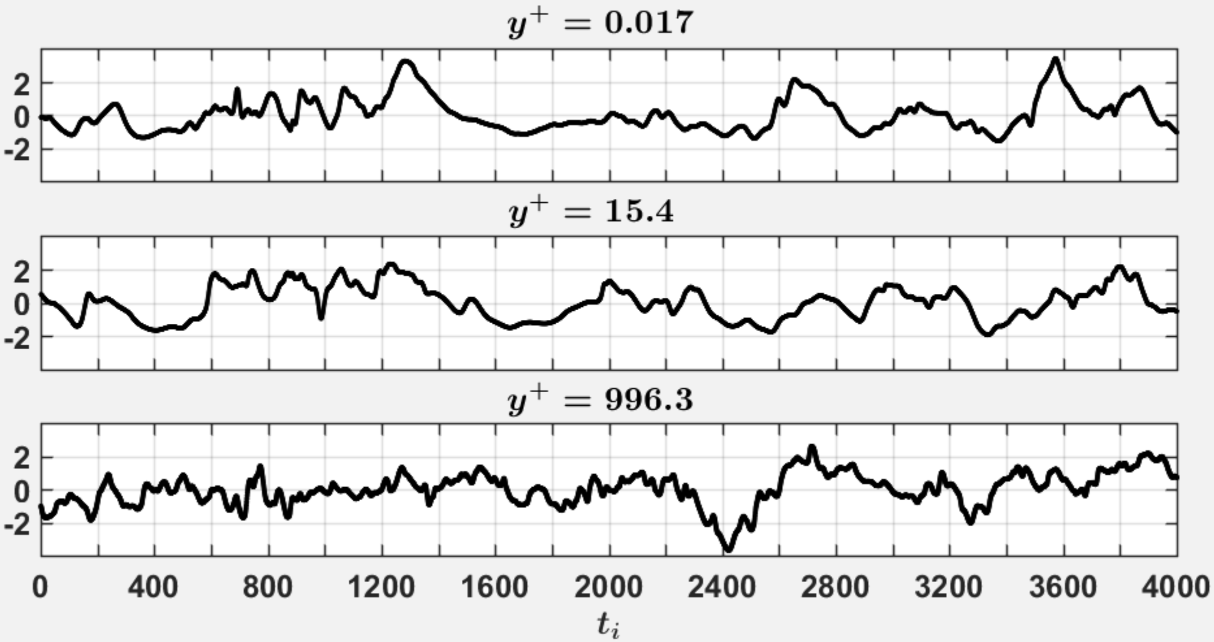
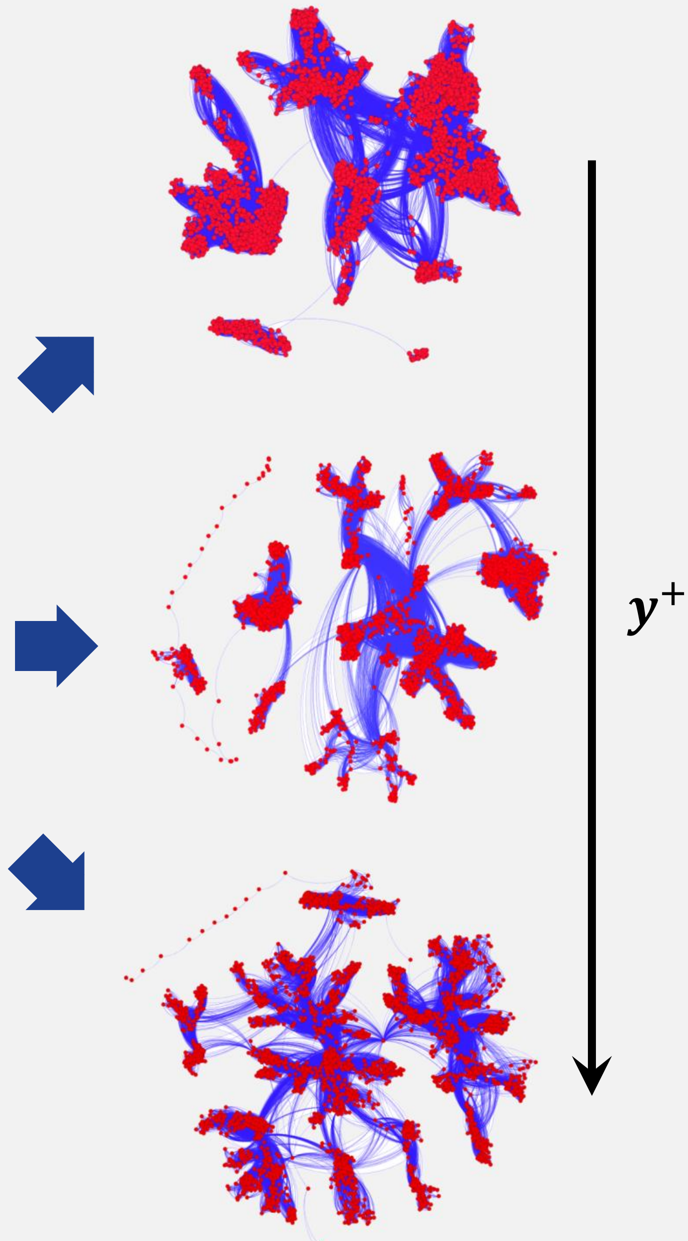
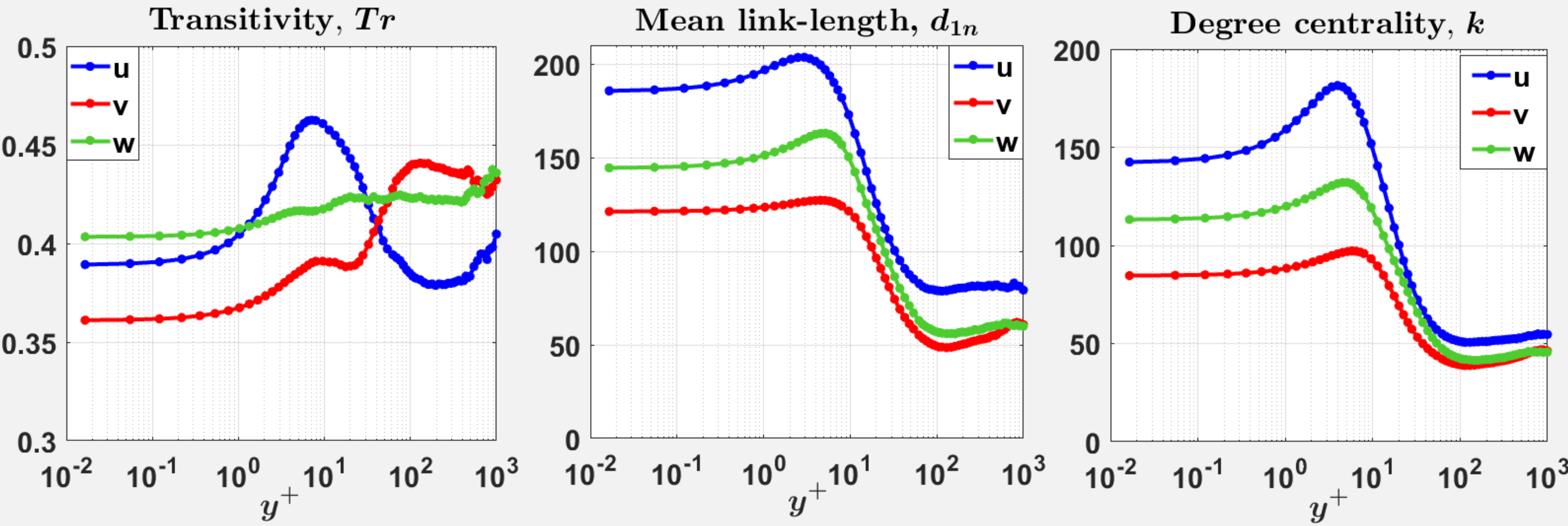




Networks inherit the temporal structure of the series

Results

Velocity components: (u^*, v^*, w^*)



Variations of peaks and irregularities can be qualitatively inferred looking at the metrics behaviors along y^+

Conclusions

Meaningful Parameters:

- Transitivity → irregularities
- Mean link-length → peaks

Main Results:

- Qualitatively infer the temporal structure of the series
- Complex networks inherit the main flow dynamics

Future works:

- Deeper analysis / Different Reynolds
- Application to other turbulent flows

Thank You for Your Attention

Questions?