11th European Turbulence Conference

Temporal dynamics of small perturbations for a two-dimensional growing wake

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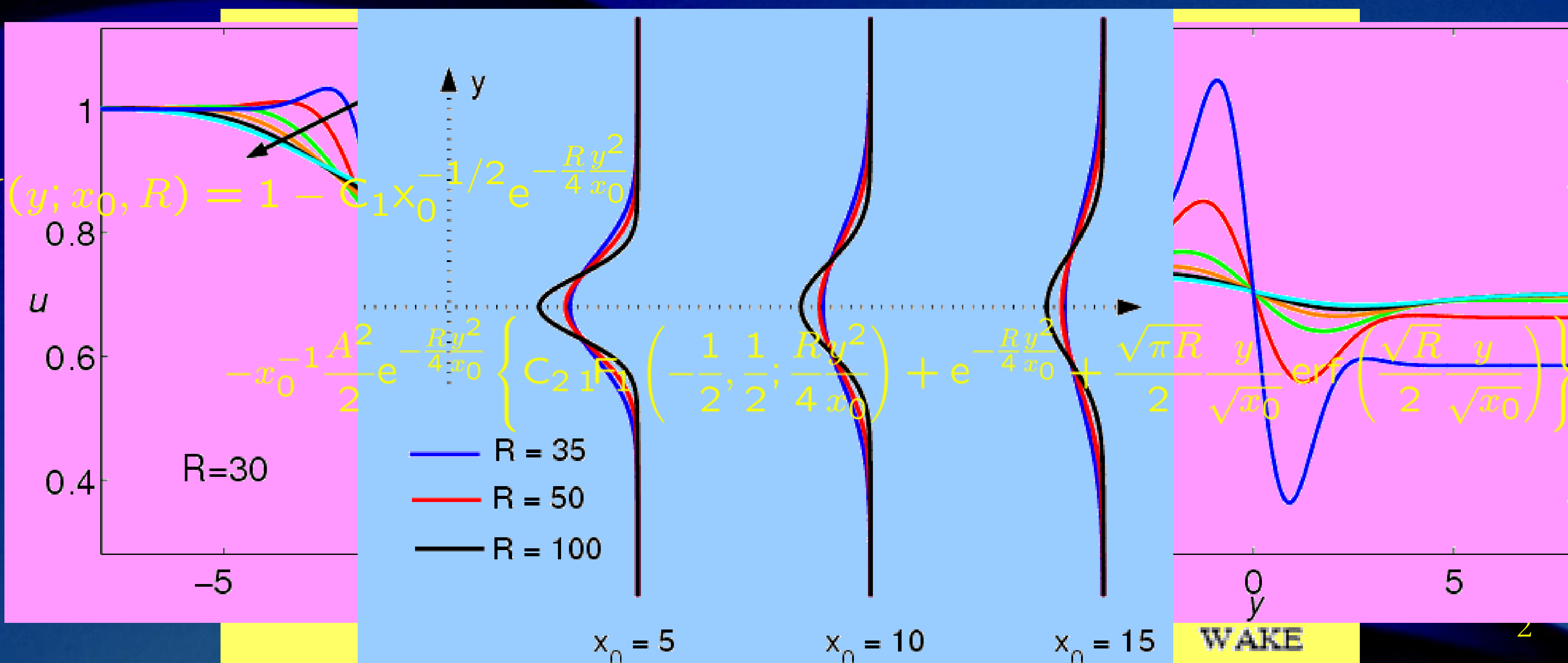
Physical problem

- Flow behind a circular cylinder ➡ steady, incompressible and viscous;
- Approximation of 2D asymptotic Navier-Stokes expansions (Belan & Tordella, 2003) ➡ parametric in x



$$U(y;x_0,R) = 1 - C_1 x_0^{-1/2} e^{-\frac{R y^2}{4 x_0}} - x_0^{-1} \frac{A^2}{2} e^{-\frac{R y^2}{4 x_0}} \left\{ C_2\, {}_1F_1\left(-\frac{1}{2}, \frac{1}{2}; \frac{R y^2}{4 x_0}\right) + e^{-\frac{R y^2}{4 x_0}} + \frac{\sqrt{\pi R}}{2} \frac{y}{\sqrt{x_0}} \operatorname{erf}\left(\frac{\sqrt{R}}{2} \frac{y}{\sqrt{x_0}}\right) \right\}$$

Formulation

- Linear, three-dimensional perturbative equations in terms of vorticity:

$$\begin{cases} \nabla^2 \tilde{v} = \tilde{\Gamma} \\ \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\tilde{\Gamma} - \frac{d^2U}{dy^2}\frac{\partial \tilde{v}}{\partial x} = \frac{1}{R}\nabla^2\tilde{\Gamma} \\ \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\tilde{\omega}_y + \frac{dU}{dy}\frac{\partial \tilde{v}}{\partial z} = \frac{1}{R}\nabla^2\tilde{\omega}_y \end{cases}$$

$$\tilde{\omega}_y = \frac{\partial \tilde{u}}{\partial z} - \frac{\partial \tilde{w}}{\partial x}$$

$$\tilde{\Gamma} = \frac{\partial \tilde{\omega}_z}{\partial x} - \frac{\partial \tilde{\omega}_x}{\partial z}$$

disturbance velocity $(\tilde{u}(t,x,y,z), \tilde{v}(t,x,y,z), \tilde{w}(t,x,y,z))$

disturbance vorticity $(\tilde{\omega}_x(t,x,y,z), \tilde{\omega}_y(t,x,y,z), \tilde{\omega}_z(t,x,y,z))$

- Moving coordinate transform $\xi = x - U_o t$ (Criminale & Drazin, 1990), $U_o = U_{y\to\infty}$

- Fourier transform in ξ and z directions: $\hat{g}(y,t;\alpha,\gamma) = \int\int_{-\infty}^{+\infty} \tilde{g} e^{i\alpha\xi + i\gamma z} d\xi dz$

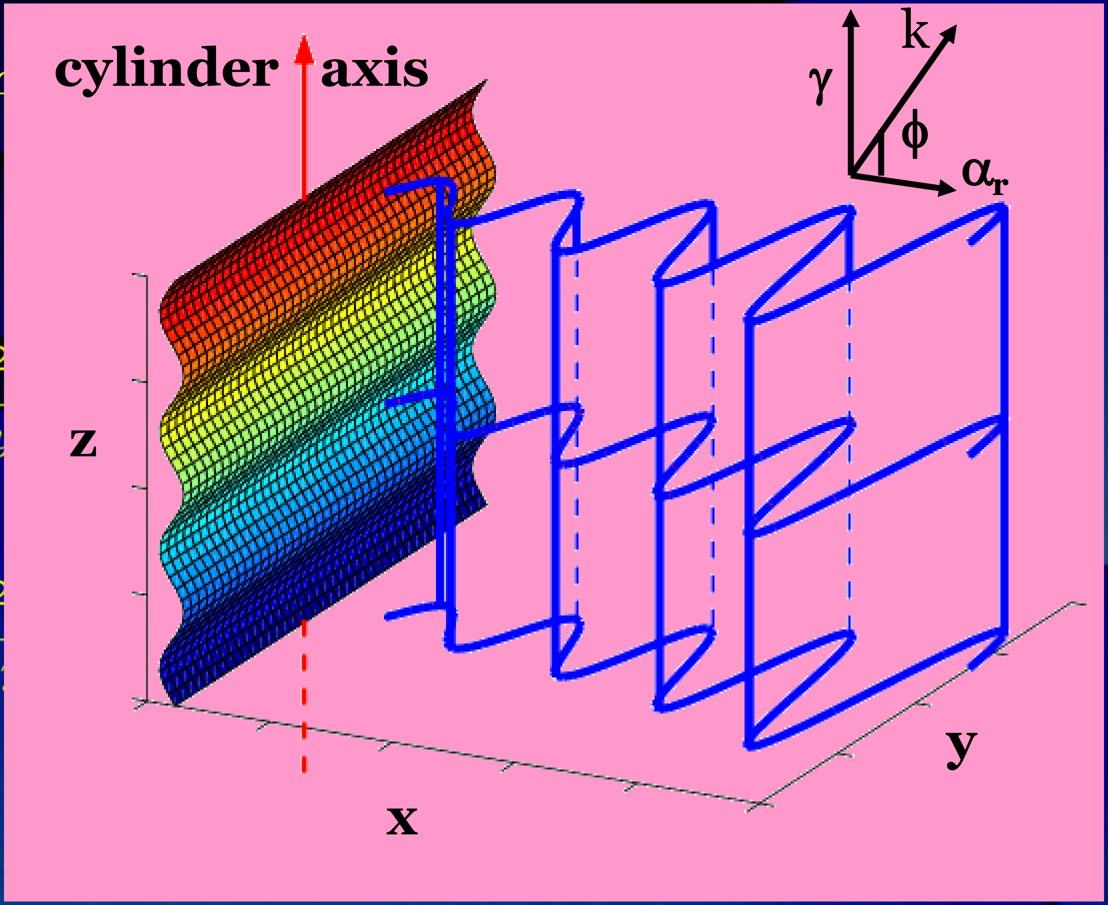
$$\frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \ldots$$

$$\frac{\partial \hat{\Gamma}}{\partial t} = -[\alpha_i - \ldots$$

$$+\frac{1}{R}[\frac{\partial^2}{\partial y \ldots}$$

$$\frac{\partial \hat{\omega}_y}{\partial t} = -[\alpha_i - \ldots$$

$$+\frac{1}{R}[\frac{\partial^2}{\partial \ldots}$$



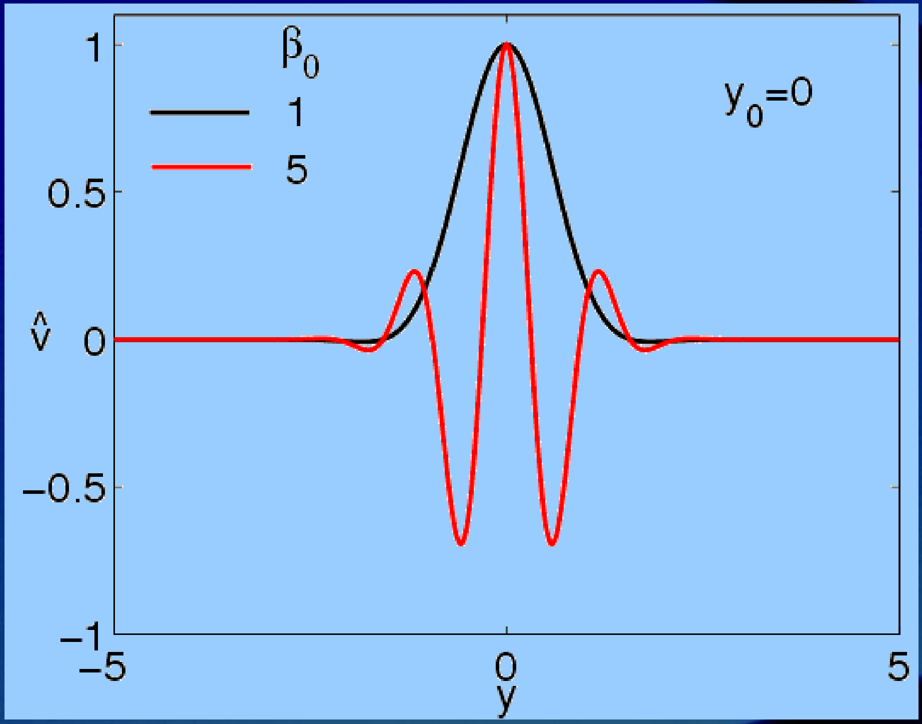
$\alpha_r = k\cos(\Phi)$ wavenumber in x-direction $\gamma = k\sin(\Phi)$ wavenumber in z-direction

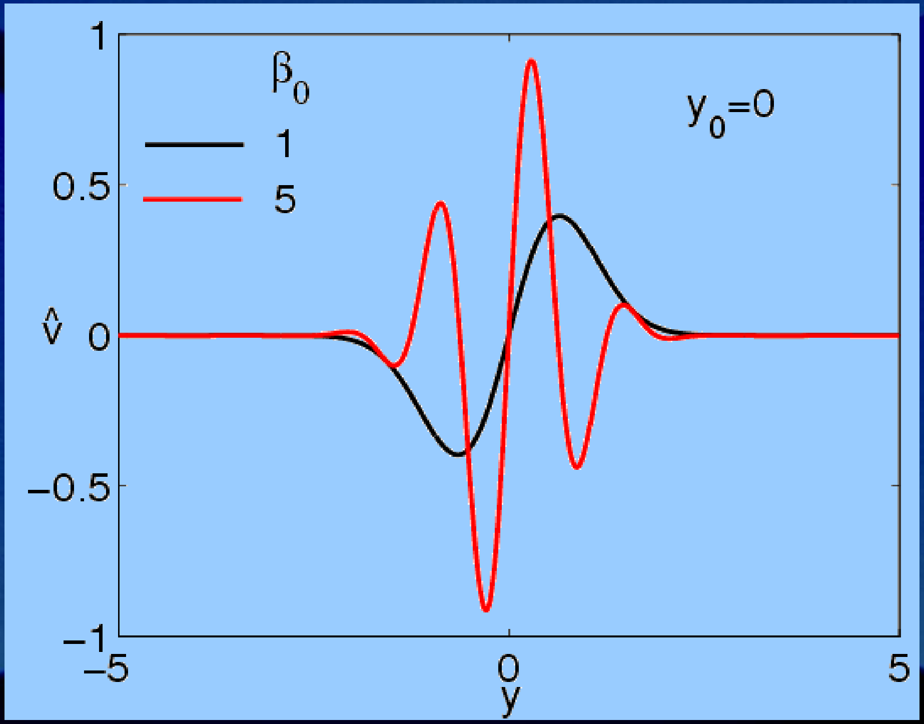
$\Phi = \tan^{-1}(\gamma/\alpha_r)$ angle of obliquity $k = (\alpha_r^2 + \gamma^2)^{1/2}$ polar wavenumber

- Initial disturbances periodic and bounded in the free stream:

$$\widehat{\omega}_y(y,t=0)=0 \begin{cases} \widehat{v}(y,t=0)=e^{-(y-y_0)^2}sin(\beta_0(y-y_0)) & \text{asymmetric} \\ \text{or} \\ \widehat{v}(y,t=0)=e^{-(y-y_0)^2}cos(\beta_0(y-y_0)) & \text{symmetric} \end{cases}$$

- Velocity field bounded in the free stream ➡ perturbation kinetic energy is finite.





Early transient and asymptotic behaviour

- Total kinetic energy *E* and kinetic energy density *e* of the perturbation

$$E(t) = \int_x \int_y \int_z \frac{1}{2}(\tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2)dxdydz$$

$$e(t;\alpha,\gamma) = k^2 E(t) = \frac{1}{2}\int_y (|\frac{\partial \hat{v}}{\partial y}|^2 + k^2|\hat{v}|^2 + |\hat{\omega}_y|^2)dy$$

- The growth function *G*

$$G(t;\alpha,\gamma) = \frac{e(t;\alpha,\gamma)}{e(t=0;\alpha,\gamma)}$$

measures the growth of the perturbation energy at time *t*.

- The temporal growth rate *r* (Lasseigne et al., 1999) is

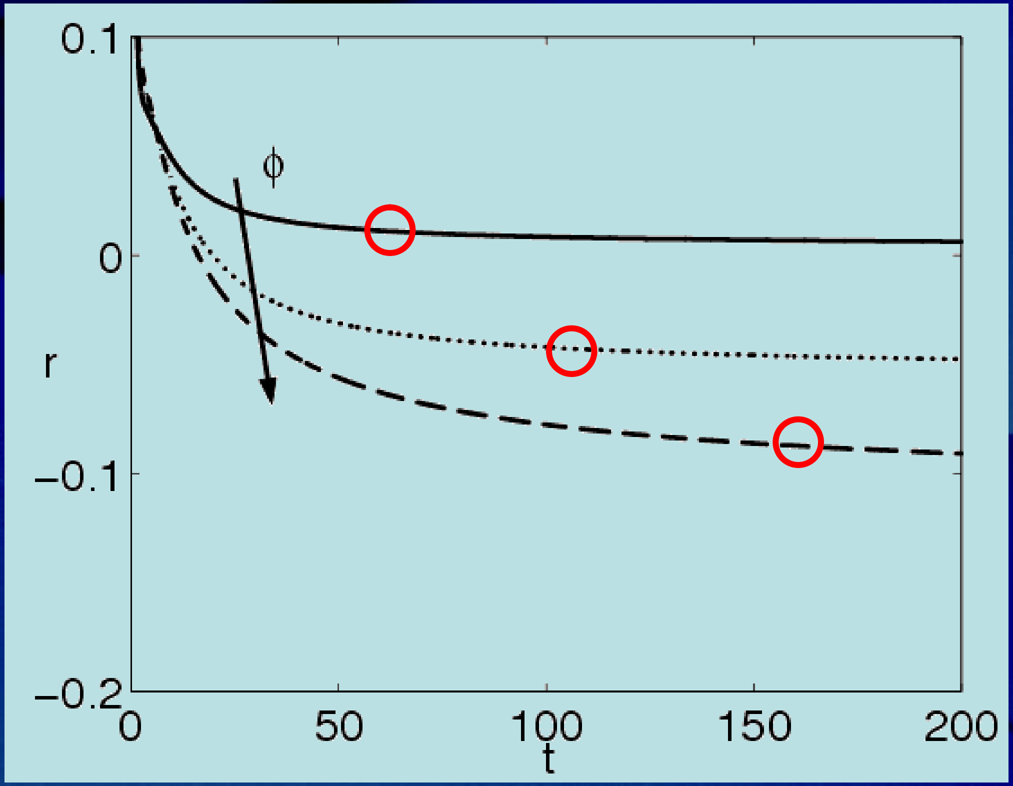
$$r(t;\alpha,\gamma) = \frac{log|e(t;\alpha,\gamma)|}{2t}$$

- The angular frequency *f* (Whitham, 1974) is

$$f(t;\alpha,\gamma) = \frac{d\varphi(t;\alpha,\gamma)}{dt}$$

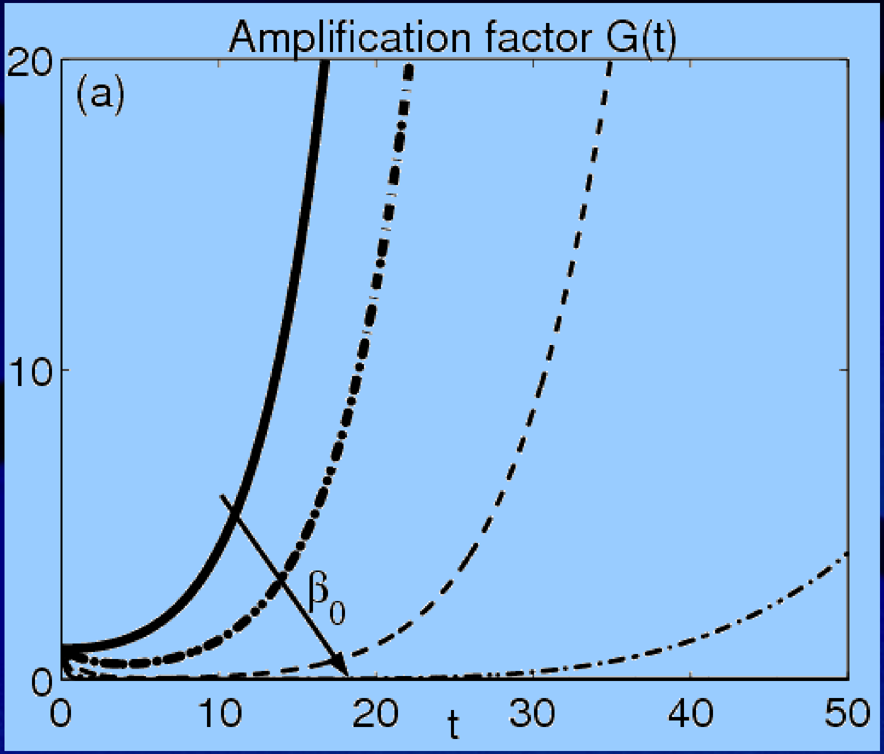
φ perturbation phase

- Asymptotic behaviour → the temporal growth rate r asymptotes to a constant value ($dr/dt < \varepsilon \sim 10^{-4}$).

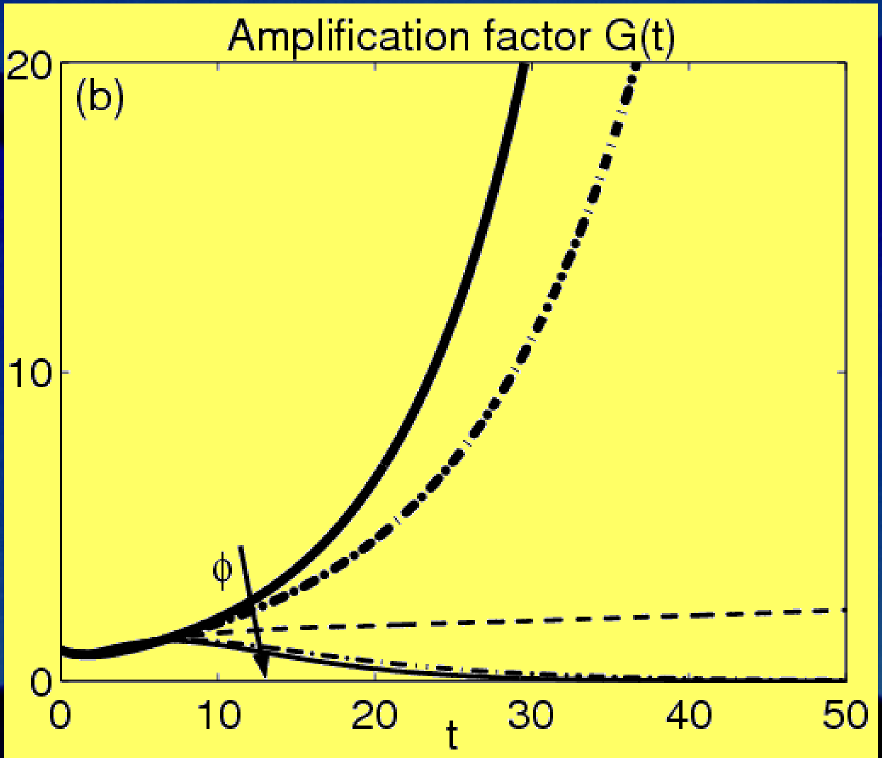


→ Comparison with normal mode theory

Results

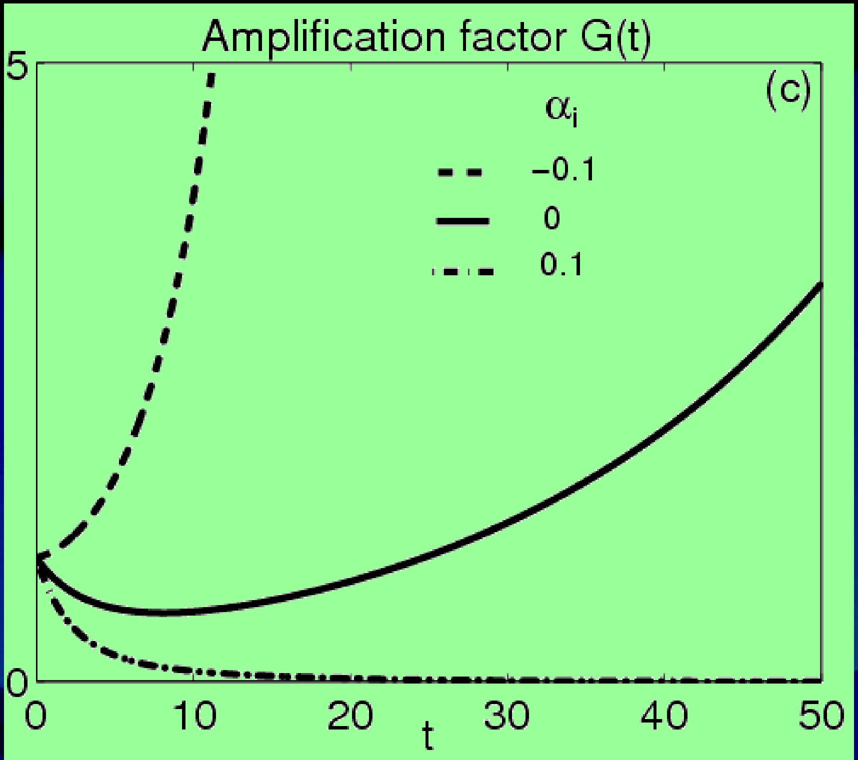


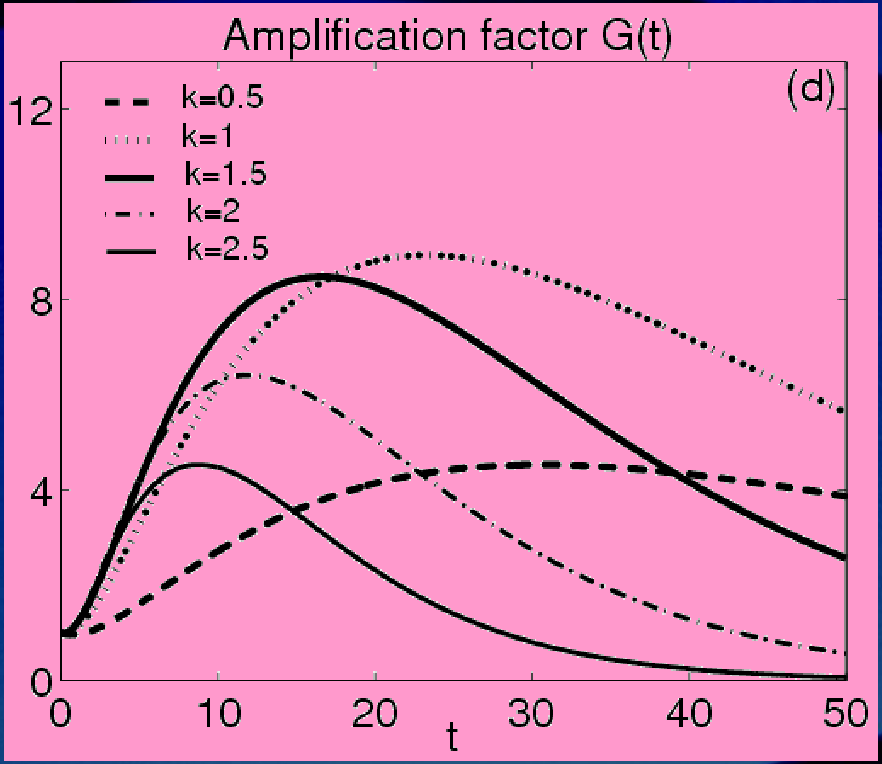
(a): $R=50$, $y_o=0$, $k=0.9$, $\alpha_i=-0.15$, $\Phi=0$, $x_o=14$, asymmetric initial condition, $\beta_o=1, 3, 5, 7$.



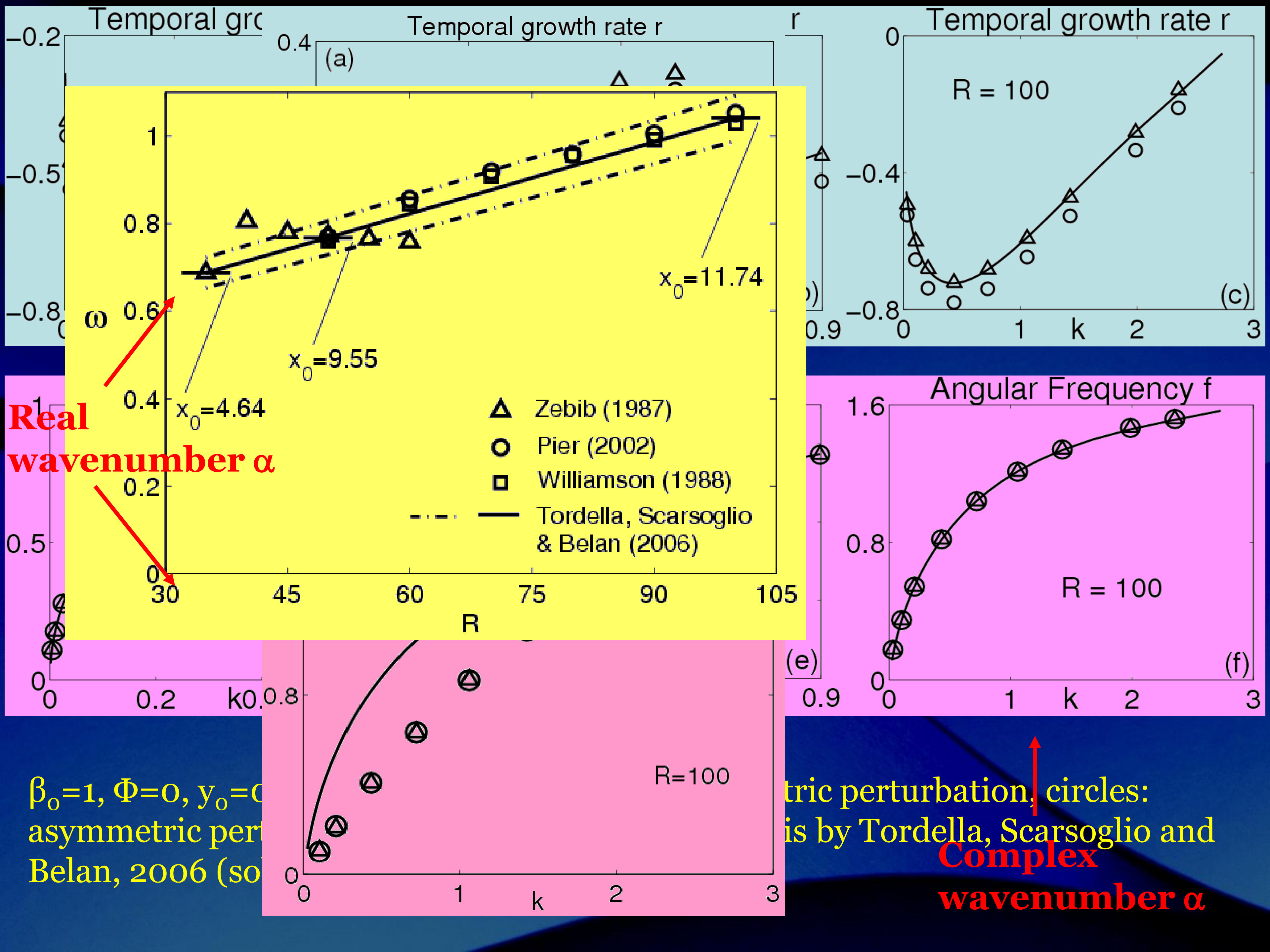
(b): $R=100$, $y_o=0$, $k=1.2$, $\alpha_i=0.1$, $\beta_o=1$, $x_o=10.15$, symmetric initial condition, $\Phi=0, \pi/8, \pi/4, (3/8)\pi, \pi/2$.

(c): R=50, y_o=0, k=0.3, β_o=1, Φ=0, x_o=5.20, symmetric initial condition,
α_i =-0.1, 0, 0.1.

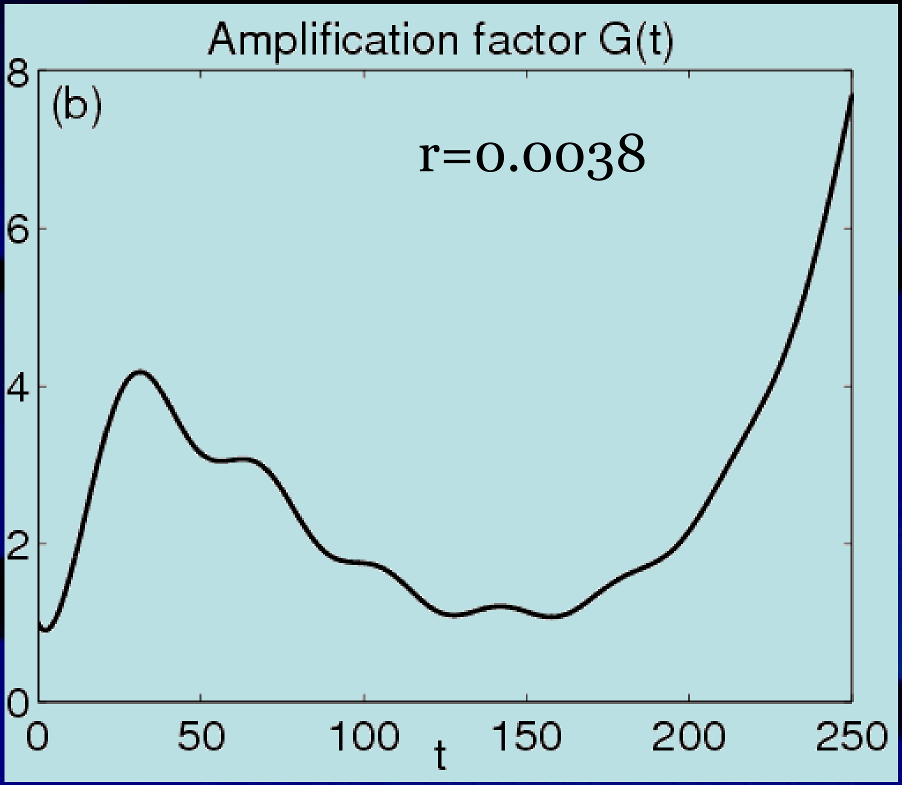




(d): R=100, y_o=0, α_i =0.01, β_o=1, Φ=π/2, x_o=7.40, symmetric initial condition, k=0.5, 1, 1.5, 2, 2.5.



Temporal growth rate r
(a)
(c)
R = 100
k
ω
R
x0=4.64
x0=9.55
x0=11.74
Zebib (1987)
Pier (2002)
Williamson (1988)
Tordella, Scarsoglio & Belan (2006)
Real wavenumber α
Angular Frequency f
(e)
(f)
R = 100
R=100
Complex wavenumber α
β0=1, Φ=0, y0=0
tric perturbation, circles:
asymmetric pert
is by Tordella, Scarsoglio and
Belan, 2006 (so



(a): R=100, y_o=0, x_o=9, k=1.7, α_i =0.05, β_o=1, symmetric initial condition, Φ=π/8.

(b): R=100, y_o=0, x_o=11, k=0.6, α_i=-0.02, β_o=1, asymmetric initial condition, Φ=π/4.

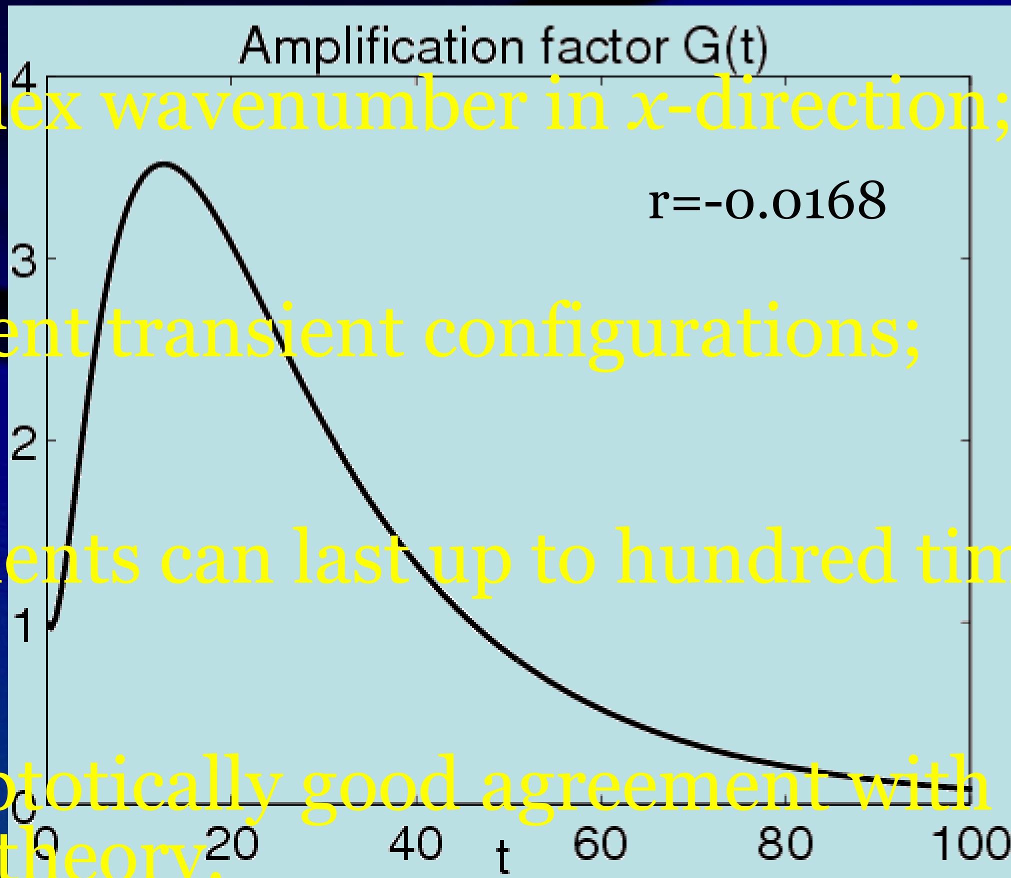
$$\hat{u} = \frac{\alpha D\hat{v} - \gamma\hat{\omega}_y}{i(\alpha^2+\gamma^2)} \qquad \hat{w} = \frac{\gamma D\hat{v} + \alpha\hat{\omega}_y}{i(\alpha^2+\gamma^2)}$$

$$\begin{cases} \bar{u}(x,y,z,t;\alpha,\gamma) \\ \bar{v}(x,y,z,t;\alpha,\gamma) \\ \bar{w}(x,y,z,t;\alpha,\gamma) \end{cases}$$

where $\bar{g}(x,y,z,t;\alpha,\gamma) = \frac{1}{2}(\hat{g}e^{-i\alpha x - i\gamma z} + \hat{g}^* e^{i\alpha^* x + i\gamma^* z})$

and $\tilde{g}(x,y,z,t) = \int_\alpha \int_\gamma \bar{g}(x,y,z,t;\alpha,\gamma)\, d\alpha\, d\gamma.$

Conclusions



- Complex wavenumber in *x*-direction;
- Different transient configurations;
- Transients can last up to hundred time scales;
- Asymptotically good agreement with normal mode theory.

R=100, y_o=0, x_o=9, k=1.7, α_i =0.05, β_o=1, symmetric initial condition, Φ=(3/8)π.