A multiscale approach to study the stability of long waves in near-parallel flows

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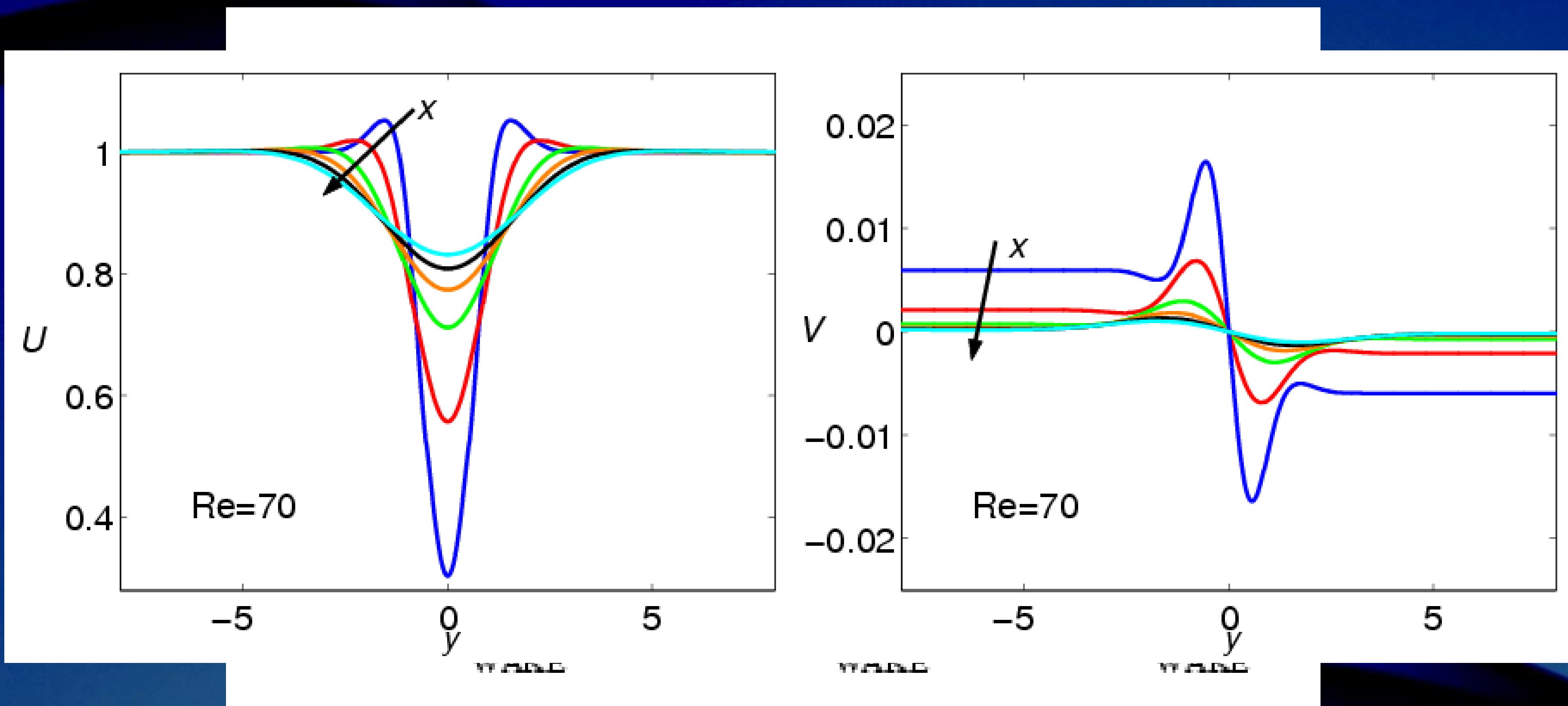
Outline

- Physical problem
- Initial-value problem
- Multiscale analysis for the stability of long waves
- Conclusions

Physical Problem

- Flow behind a circular cylinder ➡ steady, incompressible and viscous;

- Approximation of 2D asymptotic Navier-Stokes expansions (Belan & Tordella, 2003), 20≤Re≤100.



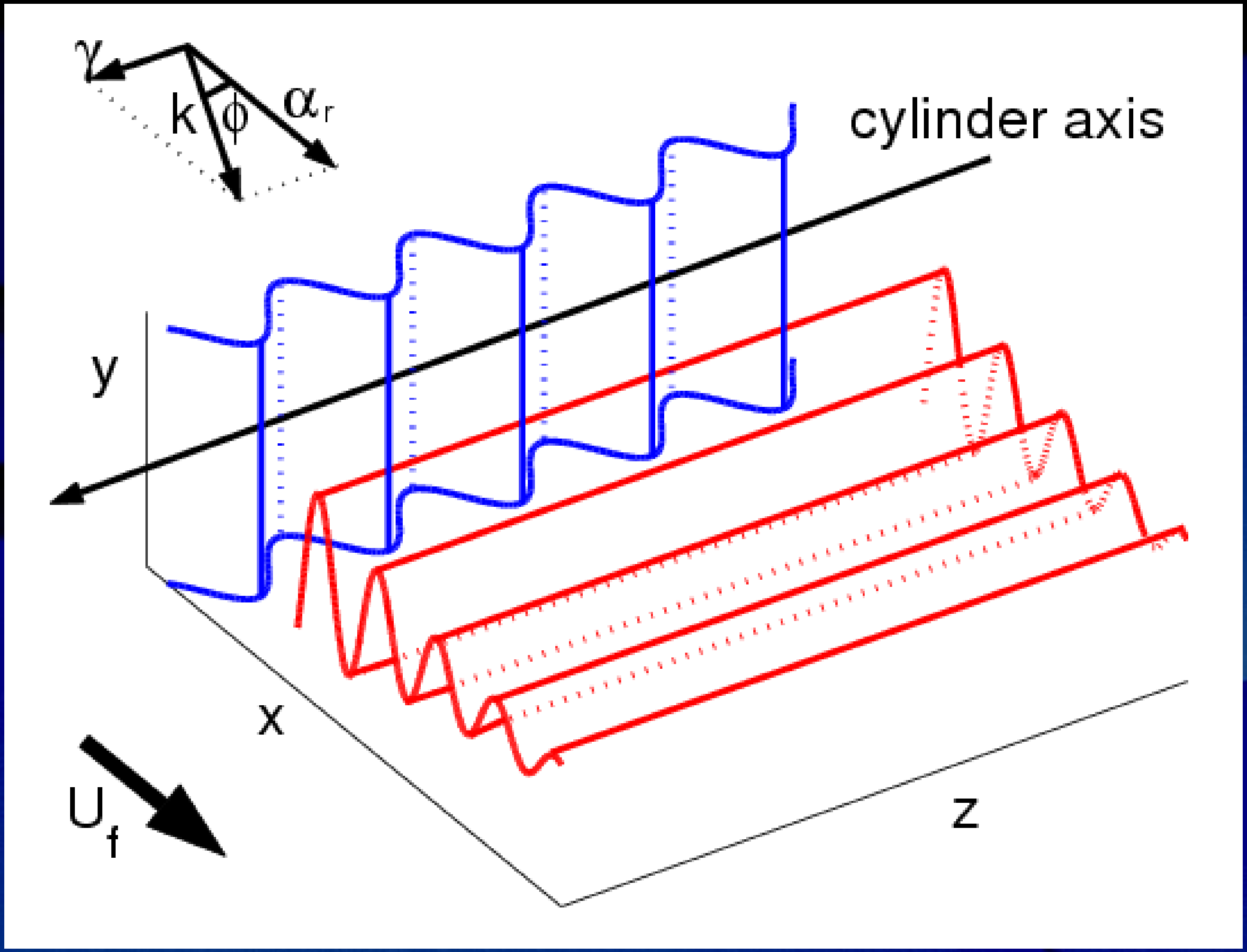
Initial-value problem

- Linear, three-dimensional perturbative equations in terms of vorticity and velocity (Criminale & Drazin, 1990);
- Base flow parametric in *x* and *Re* ➡ U(y; x_0, Re)
- Laplace-Fourier transform in x and z directions for perturbation quantities:

$$\begin{cases} \dfrac{\partial^2 \hat{v}}{\partial y^2} & - \quad (k^2 - \alpha_i^2 + 2ikcos(\phi)\alpha_i)\hat{v} = \hat{\Gamma} \\ \dfrac{\partial \hat{\Gamma}}{\partial t} = & - \quad (ikcos(\phi) - \alpha_i)U\hat{\Gamma} + (ikcos(\phi) - \alpha_i)\dfrac{d^2U}{dy^2}\hat{v} \\ & + \quad \dfrac{1}{Re}[\dfrac{\partial^2 \hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2ikcos(\phi)\alpha_i)\hat{\Gamma}] \\ \dfrac{\partial \hat{\omega}_y}{\partial t} = & - \quad (ikcos(\phi) - \alpha_i)U\hat{\omega}_y - iksin(\phi)\dfrac{dU}{dy}\hat{v} \\ & + \quad \dfrac{1}{Re}[\dfrac{\partial^2 \hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2ikcos(\phi)\alpha_i)\hat{\omega}_y] \end{cases}$$

$$\tilde{\omega}_y = \frac{\partial \tilde{u}}{\partial z} - \frac{\partial \tilde{w}}{\partial x}$$

$$\tilde{\Gamma} = \frac{\partial \tilde{\omega}_z}{\partial x} - \frac{\partial \tilde{\omega}_x}{\partial z}$$



$a_r = k\ cos(\Phi)$ wavenumber in x-direction $\gamma = k\ sin(\Phi)$ wavenumber in z-direction

$\Phi = tan^{-1}(\gamma / a_r)$ angle of obliquity $k = (a_r^2 + \gamma^2)^{1/2}$ polar wavenumber

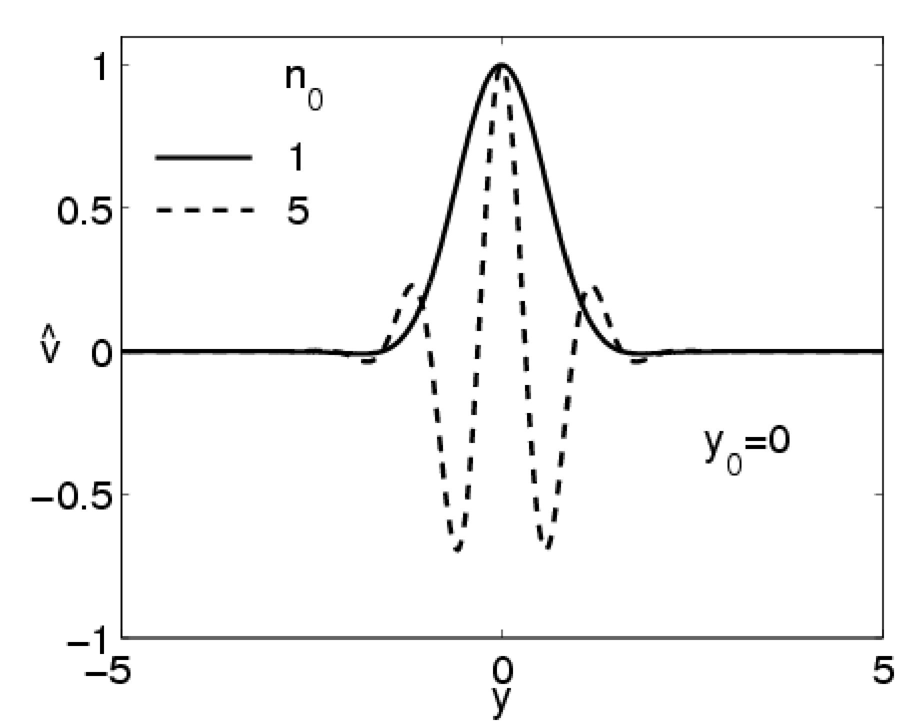
$a_i \geq 0$ spatial damping rate

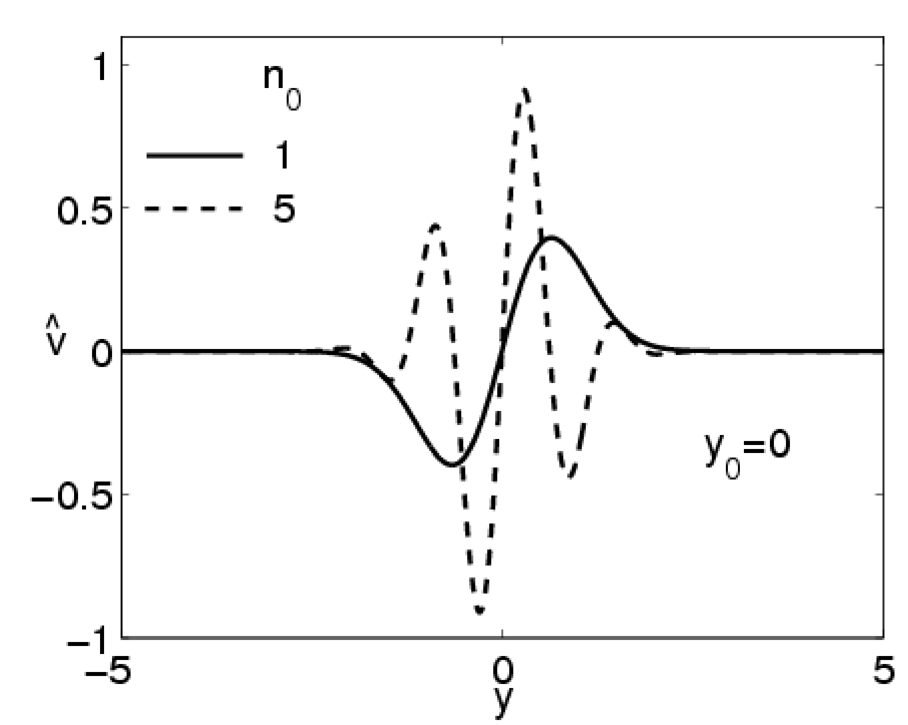
- Periodic initial conditions for $\hat{\Gamma} = \frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \alpha_i^2 + 2ikcos(\phi)\alpha_i)\hat{v}$

$$\begin{cases} \hat{v}(y, t=0) = e^{-(y-y_0)^2} cos(n_0(y-y_0)) & \text{symmetric} \\ \hat{v}(y, t=0) = e^{-(y-y_0)^2} sin(n_0(y-y_0)) & \text{asymmetric} \end{cases}$$

and $\hat{\omega}_y(y, t=0) = 0$

- Velocity field vanishing in the free stream.





Early transient and asymptotic behaviour of perturbations

- The growth function G is the normalized kinetic energy density

$$G(t;\alpha,\gamma) = \frac{e(t;\alpha,\gamma)}{e(t=0;\alpha,\gamma)}$$

and measures the growth of the perturbation energy at time t.

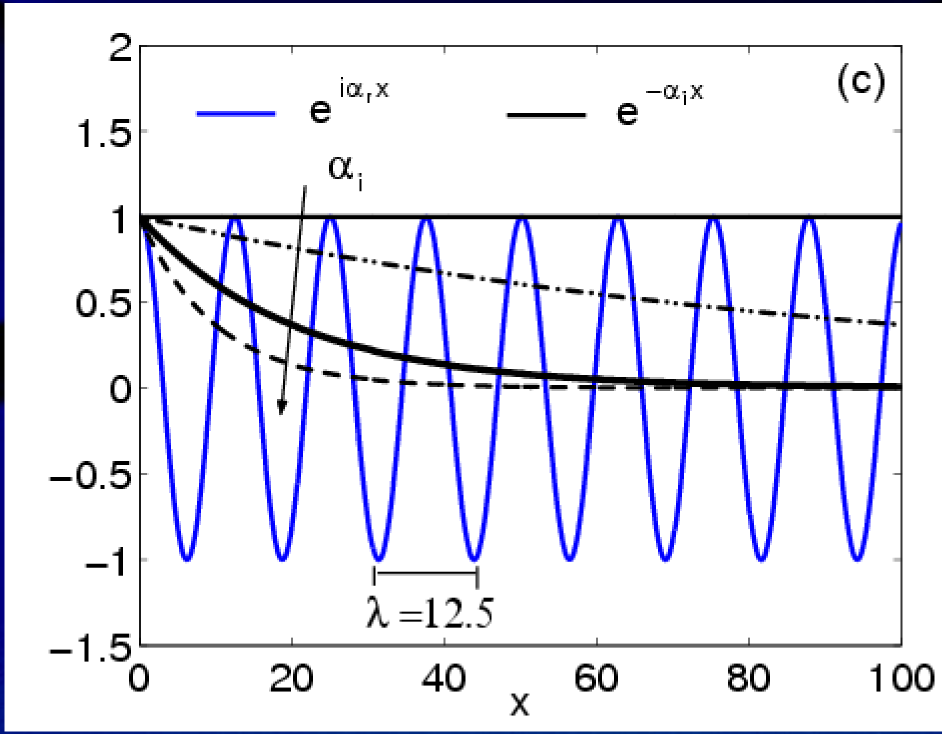
- The temporal growth rate r (Lasseigne et al., 1999) and the angular frequency ω (Whitham, 1974)

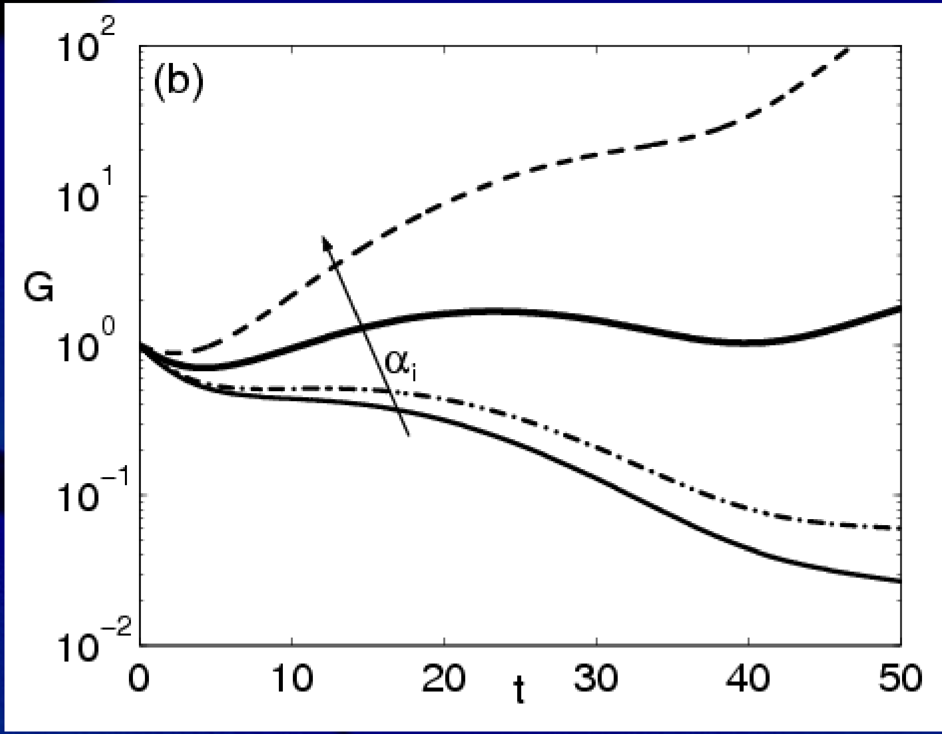
$$r(t;\alpha,\gamma) = \frac{log|e(t;\alpha,\gamma)|}{2t}, \quad t > 0$$

$$\omega(t;\alpha,\gamma) = \frac{|d\varphi(t;\alpha,\gamma)|}{dt}$$

φ perturbation phase

Exploratory analysis of the transient dynamics

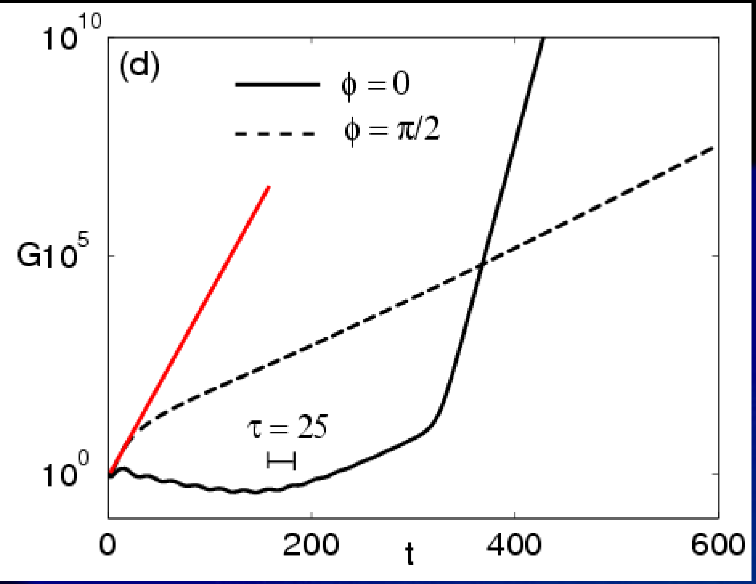




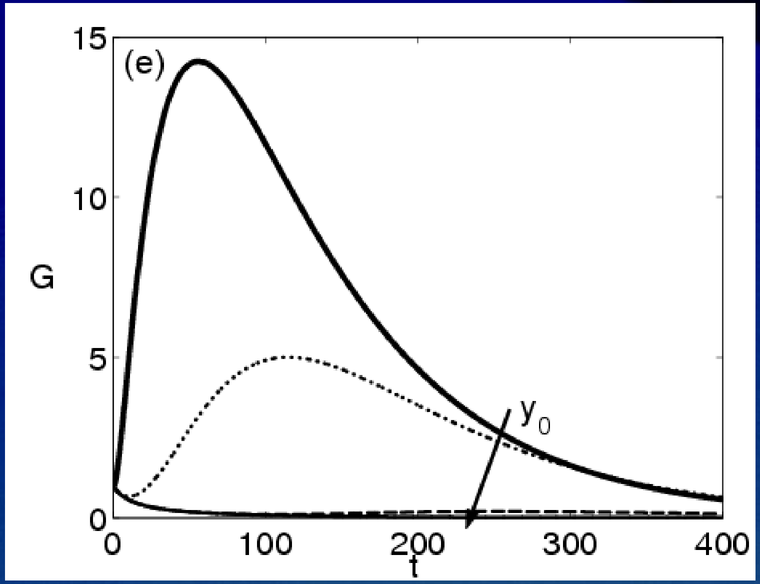
(c): Wave spatial evolution in the x direction, for symmetric, $\alpha_i=0,0.01,0.05,0.1$.

(a): R=100, $y_0=0$, $x_0=6.50$, $\alpha_i=0.01$, $n_0=1$, $k_r=0.5$, $\Phi=3/8\pi$, k=0.5,1,1.5,2,2.5.

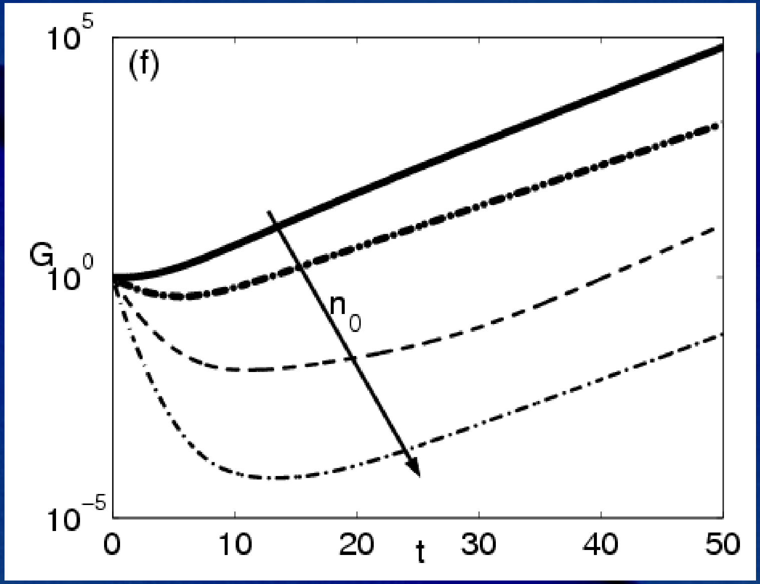
(b): R=50, $y_0=0$, $x_0=7$, k=0.5, $\Phi=0$, asymmetric, $n_0=1$, α_i = 0,0.01,0.05,0.1.



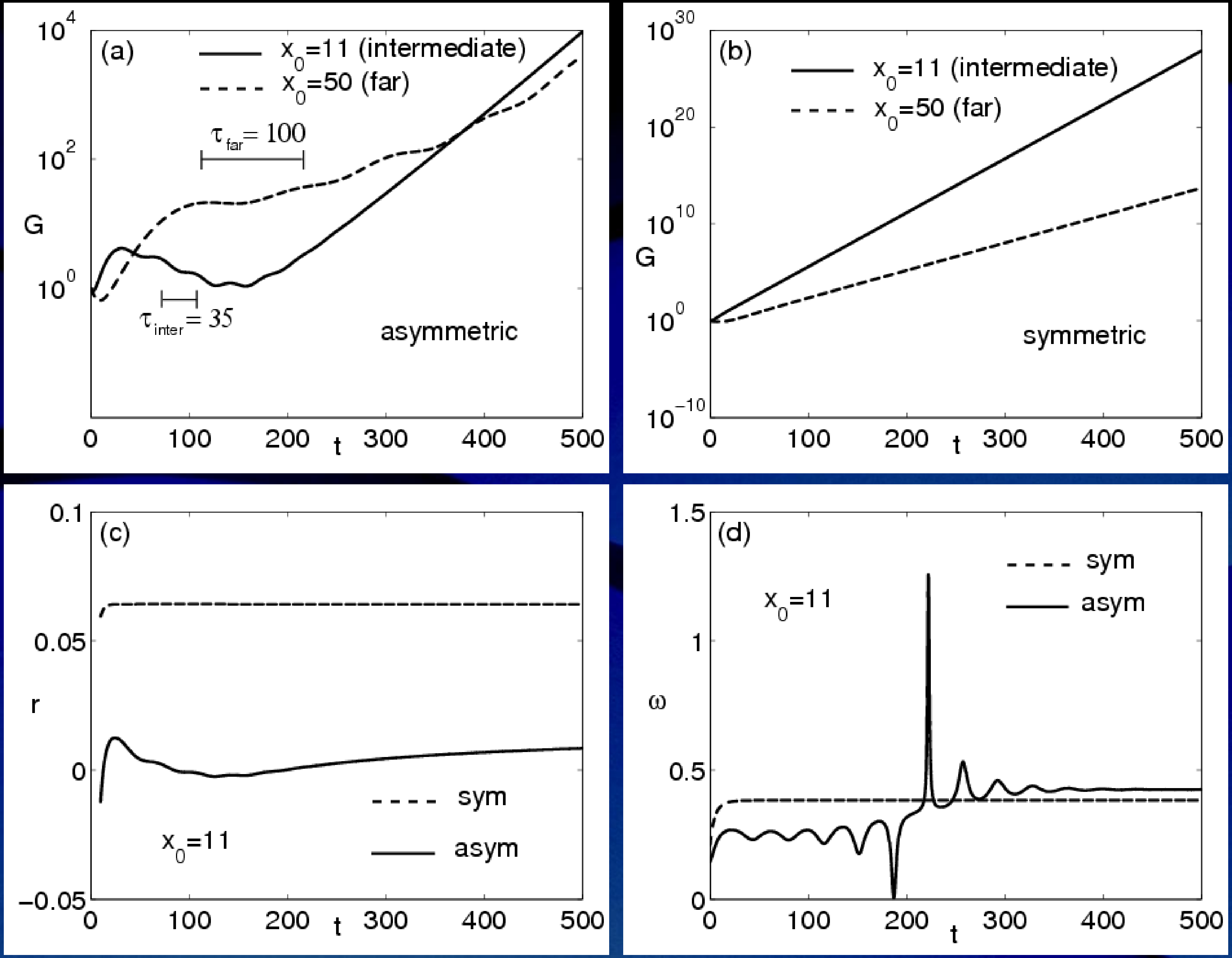
(d): R=100, y_0=0, x_0=11.50, k=0.7, asymmetric, α_i=0.02, n_0=1, Φ=0, $\pi/2$.



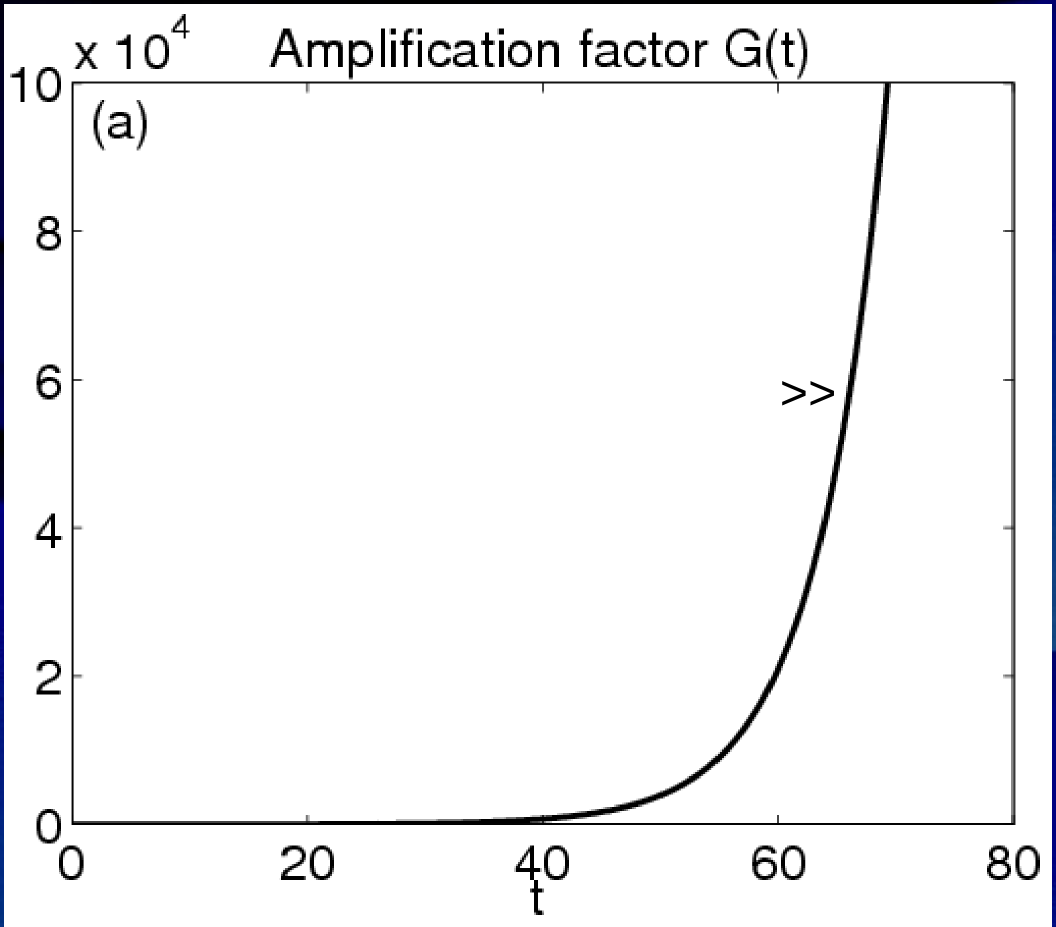
(e): R=100 x_0=12, k=1.2, α_i=0.01, symmetric, n_0=1, Φ= $\pi/2$, y_0=0,2,4,6.



(f): R=50 x_0=14, k=0.9, α_i=0.15, asymmetric, y_0=0, Φ= $\pi/2$, n_0=1,3,5,7.



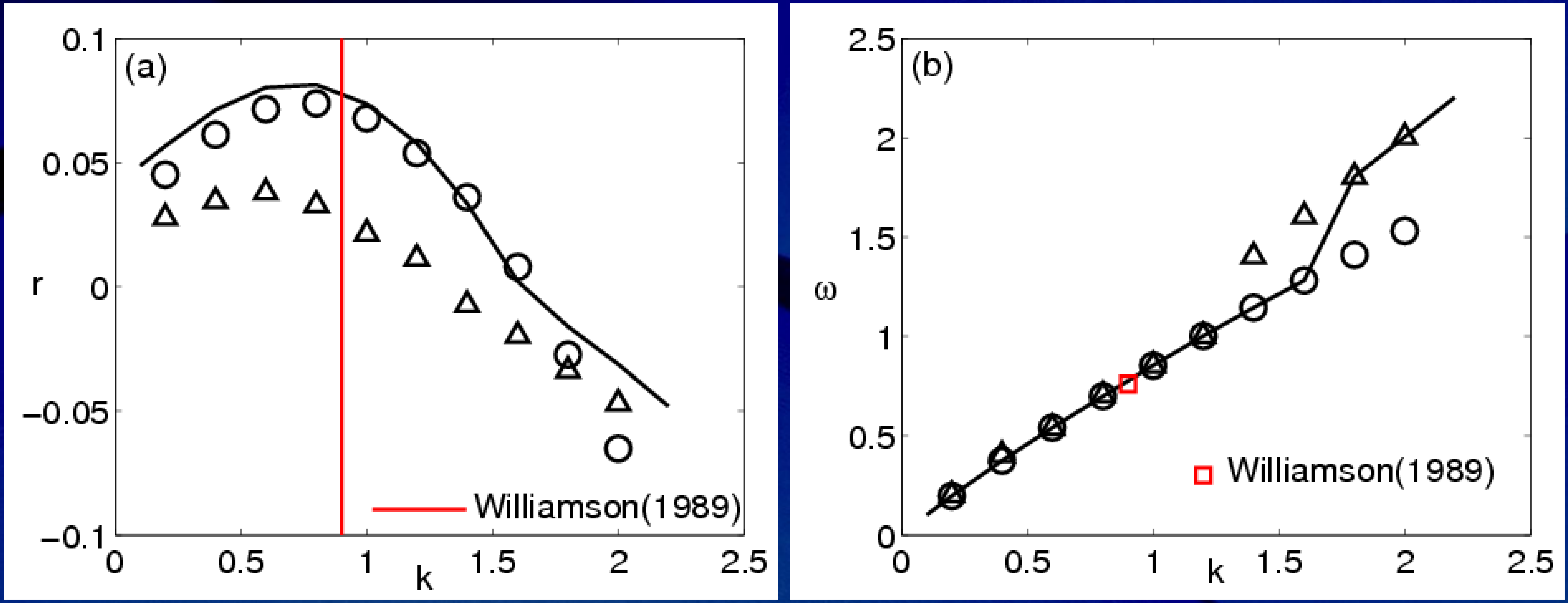
(a)-(b)-(c)-(d): R=100, y_0=0, k=0.6, α_i=0.02, n_0=1, $\Phi=\pi/4$, x_0=11 and 50, symmetric and asymmetric.



(a): R=100, y_0=0, x_0=9, k=1.7, α_i =0.05, n_0=1, symmetric, $\Phi=\pi/8$.

Asymptotic fate and comparison with modal analysis

- Asymptotic state: the temporal growth rate *r* asymptotes to a constant value ($dr/dt < \varepsilon \sim 10^{-4}$).



(a)-(b): Re=50, α_i=0.05, Φ=0, x_0=11, n_0=1, y_0=0. Initial-value problem (triangles: symmetric, circles: asymmetric), normal mode analysis (black curves), experimental data (Williamson 1989, red symbols).

Multiscale analysis for the stability of long waves

- Different scales in the stability analysis:
  - Slow scales (base flow evolution);
  - Fast scales (disturbance dynamics);
- In some flow configurations, long waves can be destabilizing (for example Blasius boundary layer and 3D cross flow boundary layer);
- In such instances the perturbation wavenumber of the unstable wave is much less than O(1).

Small parameter is the polar wavenumber of the perturbation:

$$k \ll 1$$

Full linear system

$$\begin{cases} \frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \alpha_i^2 + 2ikcos(\phi)\alpha_i)\hat{v} = \hat{\Gamma} \\ \frac{\partial \hat{\Gamma}}{\partial t} = G\hat{\Gamma} + H\hat{v} + K\hat{\omega}_y \\ \frac{\partial \hat{\omega}_y}{\partial t} = L\hat{\omega}_y + M\hat{v} \end{cases}$$

$$G = G(y; k, \phi, \alpha_i, Re)$$

base flow

(U(x,y:Re), V(x,y;Re))

Multiple scales hypothesis

- Regular perturbation scheme, k<<1:

$$\begin{aligned} \hat{v} &= \hat{v}_0 + k\hat{v}_1 + k^2\hat{v}_2 + \dots \\ \hat{\Gamma} &= \hat{\Gamma}_0 + k\hat{\Gamma}_1 + k^2\hat{\Gamma}_2 + \dots \\ \hat{\omega}_y &= \hat{\omega}_{y0} + k\hat{\omega}_{y1} + k^2\hat{\omega}_{y2} + \dots \end{aligned}$$

- Temporal scales: $t, \tau = kt, T = k^2 t;$
- Spatial scales: $y, Y = ky;$

Order O(1)

$$\left\{ \begin{array}{rcl} \dfrac{\partial^2 \widehat{v}_0}{\partial y^2} + \alpha_i^2 \widehat{v}_0 & = & \widehat{\Gamma}_0 \\ \dfrac{\partial \widehat{\Gamma}_0}{\partial t} - G_h \widehat{\Gamma}_0 - H_h \widehat{v}_0 & = & 0 \\ \dfrac{\partial \widehat{\omega}_{y0}}{\partial t} - L_h \widehat{\omega}_{y0} & = & 0 \end{array} \right.$$

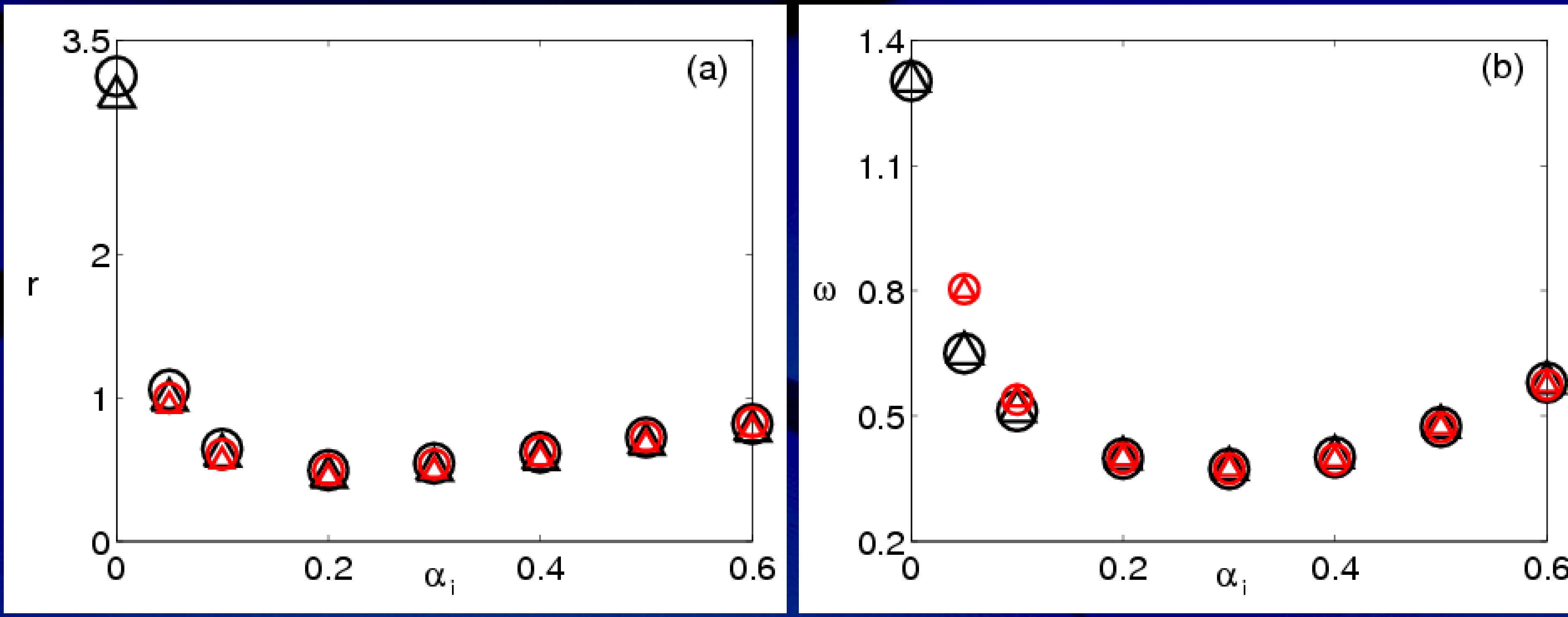
$$G_h = G_h(y; \phi, \alpha_i, Re)$$

Order O(k)

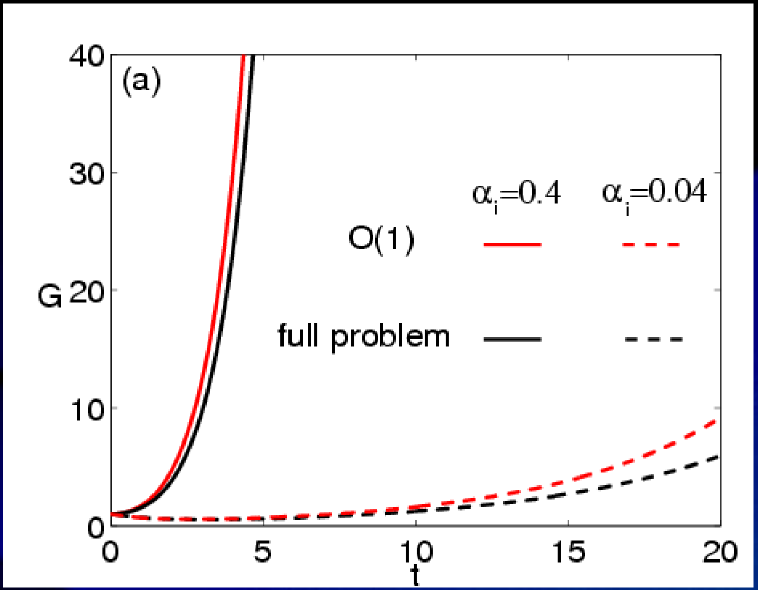
$$\left\{ \begin{array}{rcl} \dfrac{\partial^2 \widehat{v}_1}{\partial y^2} + \alpha_i^2 \widehat{v}_1 & = & -2 \dfrac{\partial^2 \widehat{v}_0}{\partial y \partial Y} + 2icos(\phi) \alpha_i \widehat{v}_0 + \widehat{\Gamma}_1 \\ \dfrac{\partial \widehat{\Gamma}_1}{\partial t} - G_h \widehat{\Gamma}_1 - H_h \widehat{v}_1 & = & -\dfrac{\partial \widehat{\Gamma}_0}{\partial \tau} + G_{h-1} \widehat{\Gamma}_0 + H_{h-1} \widehat{v}_0 + K_{h-1} \widehat{\omega}_{y0} \\ \dfrac{\partial \widehat{\omega}_{y1}}{\partial t} - L_h \widehat{\omega}_{y1} & = & -\dfrac{\partial \widehat{\omega}_{y0}}{\partial \tau} + L_{h-1} \widehat{\omega}_{y0} + M_{h-1} \widehat{v}_0 \end{array} \right.$$

$$G_{h-1} = G_{h-1}(y, Y; \phi, \alpha_i, Re)$$

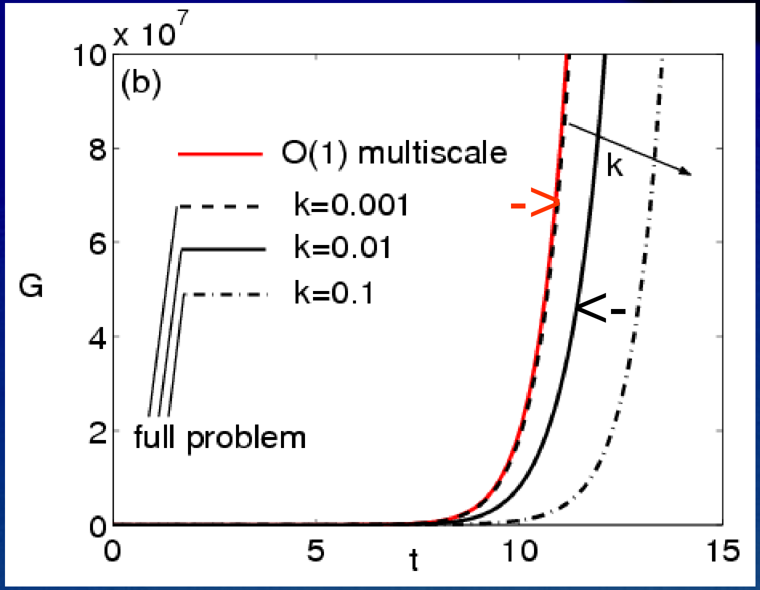
Comparison with the full linear problem

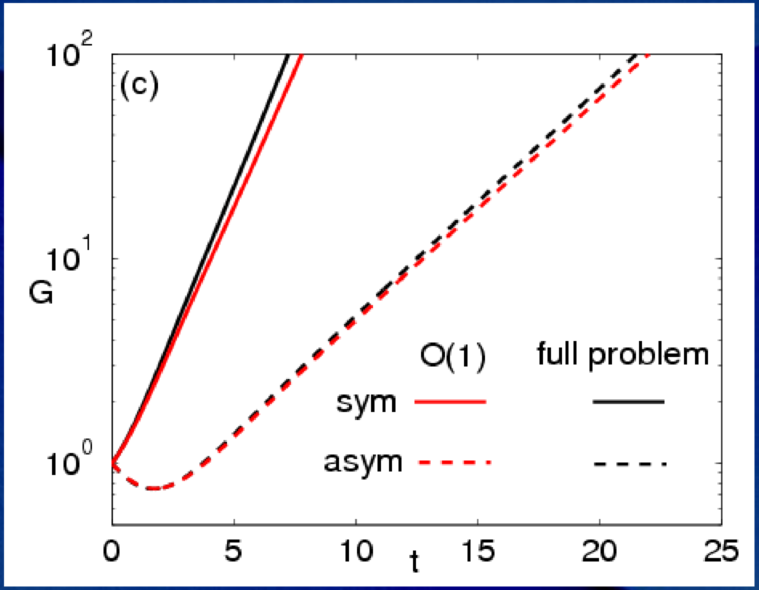


(a)-(b): Re=100, k=0.01, $\Phi=\pi/4$, $x_0=10$, $n_0=1$, $y_0=0$. Full linear problem (black circles: symmetric, black triangles: asymmetric), multiscale O(1) (red circles: symmetric, red triangles: asymmetric).



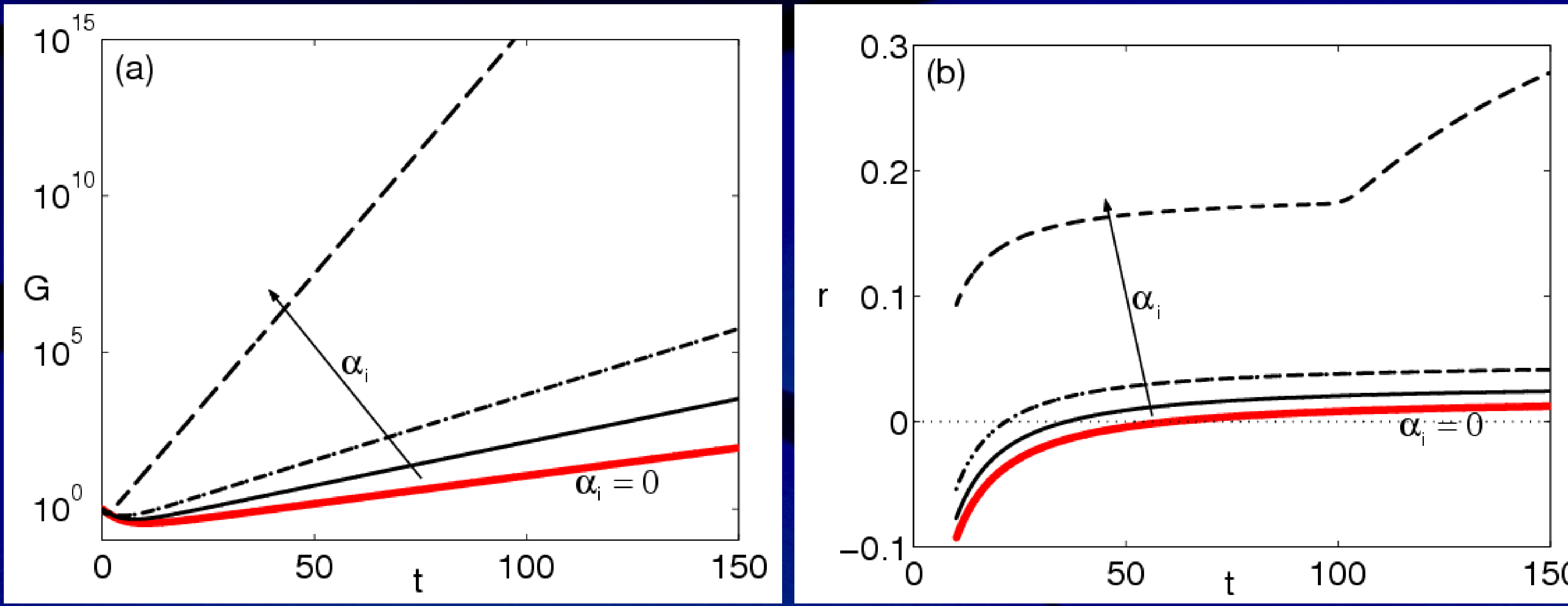
(a): R=50, y_0=0, k=0.03, n_0=1, x_0=12, Φ=π/4, asymmetric, α_i=0.04, 0.4.





(b): R=100, y_0=0, n_0=1, x_0=27, Φ=0, symmetric, α_i=0.2, k=0.1, 0.01, 0.001.

(c): R=100, y_0=0, k=0.02, x_0=13.50, n_0=1, Φ=π/2, α_i=0.08, sym and asym.



(a)-(b): $R=50$, $y_0=0$, $k=0.04$, $n_0=1$, $x_0=12$, $\Phi=\pi/2$, asymmetric, $\alpha_i=0.005$, 0.01, 0.05 (multiscale O(1)), $\alpha_i=0$ (full problem).

Conclusions

- Synthetic perturbation hypothesis (saddle point sequence);
- Absolute instability pockets (Re=50,100) found in the intermediate wake;
- Good agreement, in terms of frequency, with numerical and experimental data;
- *No information on the early time history of the perturbation;*
- Different transient growths of energy;
- Asymptotic good agreement with modal analysis and with experimental data (in terms of <u>frequency</u> and <u>wavelength</u>);
- Multiscaling O(1) for long waves well approximates full linear problem.