Multiscale analysis of long three-dimensional perturbation waves in shear flows

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In spatially developing flows different scales can be selected. In some flow configurations, it is observed that long waves can be destabilizing. An example of this behaviour is the three-dimensional cross-flow boundary layer. In such instances, when instability occurs, the perturbation wavenumber k is much less than O(1). Thus, a regular perturbation scheme can be adopted, defining as the small parameter the polar wavenumber k. Two spatial scales, a short one - y - and a long one associated to the perturbation - $Y = ky$ - can be introduced (y is the coordinate normal to the base flow direction). For the temporal dynamics, three temporal scales, the fast one - t - and the slow ones - $\tau = kt$ (perturbation) and $T = k^2 t$ (diffusion) - can be identified.

The formulation is carried on in terms of velocity and vorticity[1] by imposing initial arbitrary conditions in terms of the elements of the trigonometrical Schauder basis in the L^2 space. A combined Laplace-Fourier transform is performed[2]. We introduce the wavenumber α_r and the spatial damping $\alpha_i \geq 0$ in the evolution direction $x \geq 0$, and the wavenumber γ in the direction z normal to the base flow. The initial perturbation has a zero or positive spatial damping because its kinetic energy must be finite. Here the perturbative equations are solved up to order $O(1)$ for the case of the two-dimensional non parallel wake. The base flow is approximated using the longitudinal as well as the transversal components of an asymptotic Navier-Stokes expansion[3], so that a trace of the transversal dynamics (in particular of the entrainment) is included in the stability analysis. A section of the intermediate wake ($x_0 = 10$ body scales, region where absolute instability pockets were found by recent modal analyses [4−5]) is considered. In Fig. 1 an example of short and long term growth of the normalized kinetic energy density G of the perturbation[6] is shown. An interesting result is that, by changing the order of magnitude of α_i, perturbations that are more rapidly damped in space (Fig. 1a) lead to an immediate and monotone non decreasing growth in time (Fig. 1b).

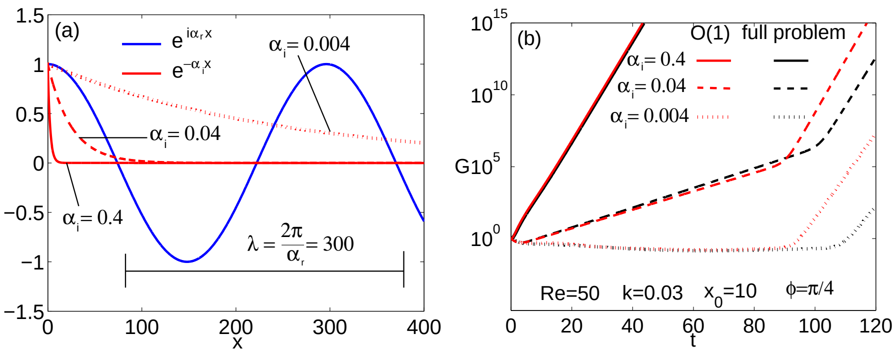

Figure 1: Effect of the spatial damping rate α_i. (a) projection of perturbation wave in the x direction and (b) normalized kinetic energy density G as function of time. Comparison between the multiscale and full problem (ϕ is the disturbance angle of obliquity with respect to the base flow plane, the inputs are asymmetric).

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