Exploring intracranial aneurysm hemodynamics with a complex networks approach

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***Abstract*—Here Complex Network theory is applied for the first time to explore the intricate hemodynamics in intracranial aneurysms. The exploratory analysis, carried out on an image-based computational hemodynamics model, suggests the formation of spatial patterns which coherently link parent vessel fluid structures to the intricate hemodynamics within the aneurysmal sac.**

***Keywords*— computational hemodynamics, complex networks, intracranial aneurysm.**

I. INTRODUCTION

In recent years coupling medical imaging and computational fluid dynamics (CFD) has shown promise for informing treatment planning and rupture risk assessment for intracranial aneurysms [1]-[5]. However, a disparity of opinions still persists in the aneurysm CFD literature regarding the diagnostic/therapeutic impact of computational hemodynamics [6], [7], where robust hemodynamic indicators of rupture risk have not yet been identified. To get more knowledge on aneurysmal hemodynamics, here an aneurysm case is considered, presenting high-frequency flow instabilities [1], [3], and on it an exploratory approach is applied, based on the complex network (CN) theory [8], [9]. CNs represent a powerful tool to explore complexity of physical systems with a huge number of interacting elements. Although interest in complex networks has been increasing in the last years, no studies have been applied to cardiovascular hemodynamics. In detail, an investigation starting from a two-point correlation for the velocity magnitude of intracranial aneurysm hemodynamics numerically solved is proposed. The analysis of the degree centrality, a well-established metric for CN characterization, suggests the formation of spatial patterns that coherently link parent vessel fluid structures to the intricate hemodynamics within the aneurysmal sac.

II. METHODS

A. CFD simulations

To explore the efficacy of complex networks when applied to cardiovascular hemodynamics, an aneurysm model from the open-source Aneurisk database (Aneurisk-Team, 2012) was selected. The Vascular Modelling ToolKit (VMTK) was used to generate a mesh of 1.8M P2-P1 tetrahedra and a pulsatile simulation was performed using a second-order accuracy, finite element CFD solver, with a temporal resolution of 20,000 time steps per cardiac cycle [1], [3]. A fully developed Womersley velocity profile was applied at the inlet, and zero pressure was specified at the outlet sections. For this CN analysis the resulting CFD data were downsampled to a 290k P1-P1 mesh and 2500 time steps.

B. Model Branch Splitting

To get insights into how local hemodynamics inside the parent vessel is correlated with fluid structure in the aneurysmal sac, the parent vessel was split into its three branches, and the aneurysmal sac was isolated. As final result of the splitting strategy, the main parent vessel (*Branch1),* its branches (*Branch2 and Branch3*), and the isolated aneurysm (*Sac*) were obtained.

C. Complex Networks: Definitions and Metrics

In graph theory, a CN is a network with significant patterns of connection between its elements and topological features that often occur when modelling real systems. A network (or graph) is defined by a set $V = 1,...,N$ of nodes and a set E of links $\{i, j\}$. In this work, we assume that the graphs are *undirected*, i.e., links have no orientation ($\{i, j\} = \{j, i\}$). In addition to that, only one link can exist between each pair of nodes. The graph is represented by the *adjacency matrix*:

$$A_{ij} = \begin{cases} 0, & if \quad \{i,j\} \notin E \\ 1 & if \quad \{i,j\} \in E \end{cases} \qquad (1)$$

A_{ij} elements are equal to 1 if a link does exist between nodes i and j, and is equal to zero elsewhere.

One of the most popular CN metrics, applied to measure the centrality of a node, is the *normalized degree centrality*:

$$k_i = \frac{1}{N-1}\sum_{j=1}^{N} A_{ij} \qquad (2)$$

that gives the number of first neighbours of node i, normalized to the total number of possible neighbours (N-1).

D. Application of CNs to the aneurysm hemodynamics

Velocity magnitude time histories along the cardiac cycle at all nodes of the discretized fluid domain were considered as obtained from the downsampled CFD simulation, and the correlation between each pair of nodes was calculated. Then, a correlation matrix R was created where each element R_{ij} is the Pearson correlation coefficient between velocity magnitude time histories in nodes i and j. The correlation matrix was used to build the network. In detail, the adjacency matrix of eq. (1) was defined by establishing that a link between nodes i and j does exist only if R_{ij} is greater than a threshold value R_t. In this way, the couples of nodes $\{i, j\}$ with $R_{ij} > R_t$ are represented in the adjacency matrix with $A_{ij} = 1$, being $A_{ij} = 0$ elsewhere. Here, we are interested in exploring the spatial patterns of correlation in the velocity field along the cardiac cycle. For this reason, correlation and adjacency matrices were built up considering: (1) the

correlation coefficients between all nodes in *Branch1* ($R^{b1\text{-}b1}$); (2) the correlation coefficients between all nodes in the *Sac* ($R^{S\text{-}S}$); (3) the correlation coefficients between nodes in *Branch1* and nodes in the *Sac* ($R^{b1\text{-}S}$).

Here the median of the distribution of the correlation coefficients $R^{b1\text{-}b1}$ between nodes in *Branch1* (Fig. 1) was selected as threshold value R_t and applied to build up the adjacency matrices of $R^{b1\text{-}b1}$, $R^{S\text{-}S}$ and $R^{b1\text{-}S}$ distributions, respectively. The obtained adjacency matrices were used to calculate the normalized degree centrality metric as in eq. (2), indicating with $k_i^{b1\text{-}S}$ the number of nodes inside the *Sac* connected to the node *i* in *Branch1*, and with $k_i^{b1\text{-}b1}$ ($k_i^{S\text{-}S}$) the number of first neighbors, inside *Branch1* (*Sac)*. The CN-based characterization of the intracranial aneurysm hemodynamics was enriched by the calculation of the entropy *H* of the normalized degree centrality k_i, defined as:

$$H = \sum_{i=1}^{N} p(k_i) \ln\left(p(k_i)\right) \qquad (3)$$

where $p(k_i)$ is the probability of k_i. The value of *H* provides a lower bound for the expected degree of centrality among velocity magnitude time histories required to represent relationships involving dependence, as sampled from $p(k)$.

III. RESULTS

The distributions of $R^{b1\text{-}b1}$, $R^{S\text{-}S}$ and $R^{b1\text{-}S}$ are presented in Fig. 1. It can be noticed that the distributions are all left-skewed, and correlation are mostly positive. The median value of the two-points correlations of velocity magnitude time histories in the *Sac* region is lower than *Branch1*. The median value of $R^{b1\text{-}b1}$was selected here as threshold value (R_t = 0.928) to calculate normalized degree centrality values.

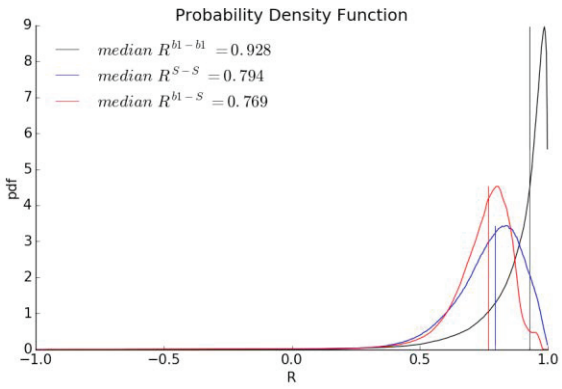

Figure 1 - Probability density functions of $R^{b1\text{-}b1}$, $R^{S\text{-}S}$ and $R^{b1\text{-}S}$. Median values of the distributions are also presented.

A visual inspection of the map of $k_i^{b1\text{-}S}$ values in the parent vessel (Fig. 2, upper panel) highlights that the velocity magnitude time histories with the highest number of links (in terms of correlation) with time histories in the aneurysmal sac are located at the outer walls of bending segments of the parent vessel, where flow instability onset has been observed [4]. The visualization of the map of degree centrality also highlights the presence of a wide region of the sac where velocity magnitude time histories are highly linked with the velocity magnitude of nodes in the sac itself ($k_i^{S\text{-}S}$ map in Fig. 2, lower panel). Entropy of normalized degree centrality in *Branch1* (H = 8.48) is higher than the *Sac* (H = 6.12). This can be interpreted as follows: a lower bound for the degree of centrality is required to represent patterns of linked (i.e., highly correlated) velocity magnitude time histories within the *Sac* than in *Branch1*. This is to say that the more the velocity magnitude time histories in the nodes of the flow field are linked (as in the *Sac*, with respect to *Branch1*), the more you can compress its representation, with implications for flow structures clustering and visualization purposes.

IV. CONCLUSION

Here we present for the first time the application of CN theory to cardiovascular hemodynamics. In detail, networks were built from spatio-temporal data following a two-point correlation approach. High degree centrality regions evidenced spatial patterns coherently moving, e.g., from the parent vessel to the aneurysmal sac and in the sac and within the sac itself. Based on present findings, the application of CNs to intricate cardiovascular flows looks promising and deserves additional future investigation.

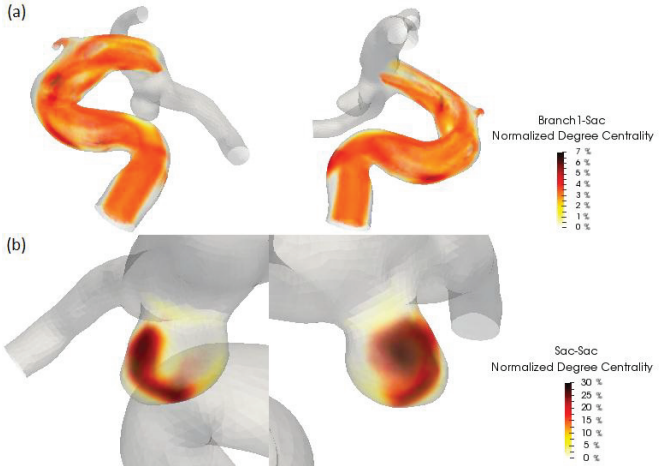

Figure 2 - upper panel: visualization (two different views) of $k^{b1\text{-}S}$ in the parent vessel; lower panel: visualization (two different views) of $k^{S\text{-}S}$ in the aneurysmal sac.

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