Tensor decomposition via Bertini software: Exercises

Elena Angelini

Bertini is a software for Numerical Algebraic Geometry, created for solving systems of polynomial equations. For an overview on Bertini, visit the website `https://bertini.nd.edu/` and download the software at the webpage `https://bertini.nd.edu/download.html`.

In this tutorial, Bertini is mainly used to numerically decomposing tensors, together with Matlab. For an example of code, download and uncompress the archive `http://web.math.unifi.it/users/angelini/Example Bertini ternary forms 334.rar`.

Exercise 1

Check numerically *Hilbert's theorem*:

"the general ternary quintic $p \in \mathbf{C}[x_0, x_1, x_2]_5$ of generic rank 7 is Waring identifiable over $\mathbf{C}$".

Exercise 2

Check numerically *Sylvester's pentahedral theorem*:

"the general quaternary cubic $p \in \mathbf{C}[x_0, x_1, x_2, x_3]_3$ of generic rank 5 is Waring identifiable over $\mathbf{C}$".

Exercise 3

Check numerically *Roberts' theorem*:

"the general polynomial vector $q = (p_1, p_2)$, with $p_1 \in \mathbf{C}[x_0, x_1, x_2]_2$ and $p_2 \in \mathbf{C}[x_0, x_1, x_2]_3$ of generic rank 4 is Waring identifiable over $\mathbf{C}$".

Exercise 4

According to *London's theorem*, 3 general ternary cubics have 2 simultaneous Waring decompositions with generic rank 6 over $\mathbf{C}$. After verifying numerically this statement, prove that:

1. there exists a non-trivial Euclidean open subset of $(\mathbf{R}[x_0, x_1, x_2]_3)^{\oplus 3}$ where 6-Waring identifiability holds over $\mathbf{R}$;

2. there exists a non-trivial Euclidean open subset of $(\mathbf{R}[x_0, x_1, x_2]_3)^{\oplus 3}$ where 6-Waring identifiability fails over $\mathbf{R}$.

Exercise 5

By adapting the technique under discussion, prove, from a numerical point of view, that the general tensor of type $(2, 2, 2, 3)$ and generic rank 4 is identifiable over $\mathbf{C}$.